A new principle for protection from seismic waves

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ABSTRACT: A notion of the wave barrier against seismic surface Love and Rayleigh-Lamb waves propagating in anisotropic layered media is introduced. Methods based on various theorems describing conditions of propagation for surface acoustic waves are analyzed. A theorem on non-existence of the “forbidden” directions for Rayleigh waves is discussed. Some engineering methods and solutions for modifying surface layers and creating seismic barriers are considered.

1 INTRODICTION

1.1 *Methods of seismic protection*

Generally, current approaches for preventing failure of structures due to seismic activity can be divided into two groups:

(i) methods for creating seismically stable structures and joints; this group contains methods of both active and passive protection;

(ii) methods for creating a kind of seismic barriers preventing seismic waves to transmit energy into the protected regions (passive).

Herein, we consider methods of seismic protection belonging to the second group.

1.2 *Types of wave barriers*

While the first group contains a lot of different engineering approaches and solutions, the second one consists in two different approaches:

(a) Transverse barriers, immersed in earth to prevent a surface wave to penetrate the protected region; see Fig.1.

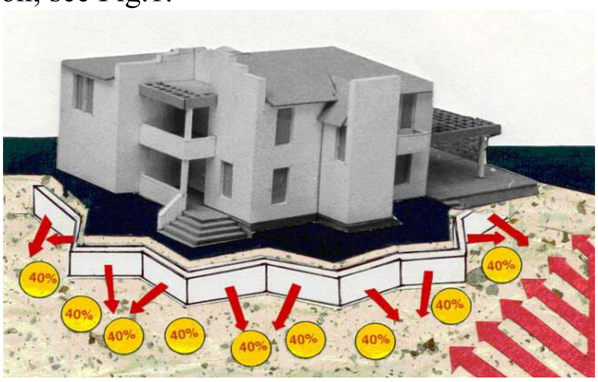

Figure 1. Transverse barrier by the Kalmatron Corporation.

To analyze effectiveness of such a barrier we should consider the typical surface seismic wave length. This length depends upon the wave angular frequency and the phase speed, but for the most dangerous seismic frequencies lying in a range 5 ÷ 15 Hz and for Rayleigh wave speeds 900 ÷ 2500m/sec, the corresponding wave lengths become 60 ÷ 500m. Taking into account this interval of the wave lengths, it is doubtful that a relatively shallow and narrow barrier shown on Fig.1 can prevent the seismic wave from penetrating the protected region. Our observation revealed that a wave having a considerably larger wavelength that the transverse barrier, actually does not “notice” a small barrier; see Fig. 2, where a simple FEM model demonstrates that a small transverse grove cannot reflect or scatter the wave.

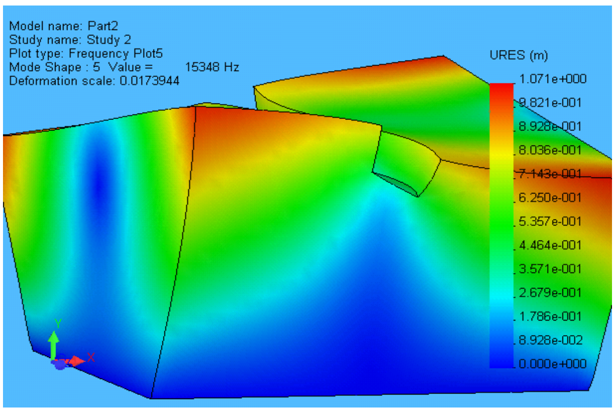

Figure 2. Interaction of a transverse grove with Rayleigh wave.

(b) Longitudinal barriers that consist of a surface layer with special physical properties that prohibit propagation of the desired surface wave; see Fig.3.

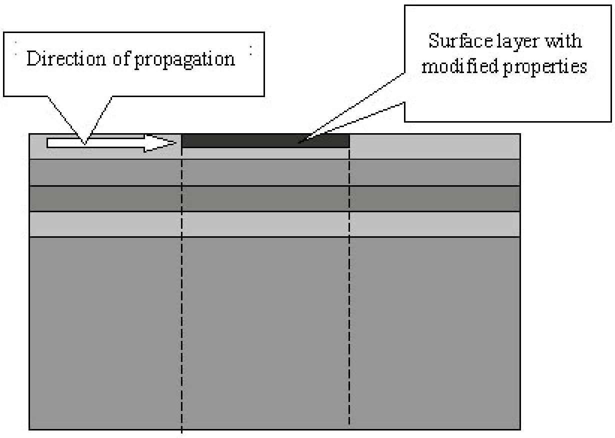

Figure 3. Longitudinal wave barrier.

One interesting approach is to create a "rough" outer surface at the upper layer (or half-space) to force the surface wave scatter by caves and swellings; see Fig.4, where a half-space with the sinusoidal roughness is pictured. In this respect, the rough surface apparently transforms the elastic half-space into viscoelastic.

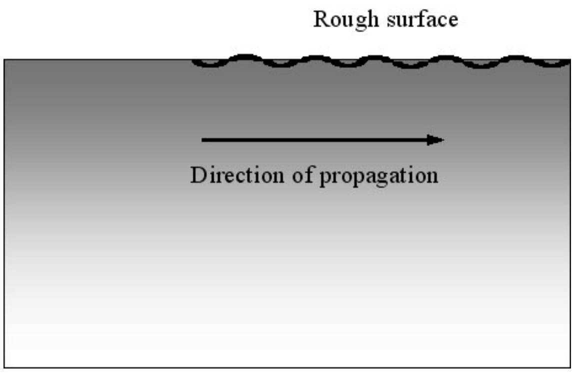

Figure 4. Half-space with rough surface

Problems of attenuation of Rayleigh waves due to roughness of the surface are discussed in quite a lot of publications (Sobczyk 1965, Urazakov & Falkovskij 1972, Maradudin & Mills 1976, Maradudin & Shen 1980, Goldstein & Lewandowsky 1990). In the latter paper change of the phase speed along with attenuation of Rayleigh wave due to roughness was observed. Actually, to achieve the desired attenuation, the period of surface imperfections should be almost equal to the period of the surface wave, and the longitudinal length of the rough surface should be much larger than the wave length.

In practice, such a rough surface can be achieved by a series of rather deep trenches oriented transversally to the most probable direction of the wave front. Some of obvious deficiencies of this method are: (i) its inability to persist the surface waves other than Rayleigh waves; (ii) protection from Rayleigh waves traveling only in directions that are almost orthogonal to orientation of the trenches; and, (iii) high sensitivity to the frequency of traveling Rayleigh waves.

1.3 *Modification of the surface layer*

Our current research is devoted to analyzing longitudinal wave barriers, as shown on Fig.3, by finding properties of the outer layer that can prevent the surface wave to propagate.

In practice, modifying physical properties of the outer layer can be achieved by reinforcing ground with piles or "soil nails"; see papers where reinforcing was studied for increasing bearing load of the soil (Blondeau 1972, 1989, De Buhan et al. 1989, Abu-Hejleh et al. 2002, Eiksund 2004, Herle 2006).

If distance between piles is sufficiently smaller than the wave length, then a reinforced region can be considered as macroscopically homogeneous and either transversely isotropic or orthotropic depending upon arrangement of piles. Of course, homogenized physical properties of the reinforced medium depend upon material of piles and distance and arrangements between them.

For stochastically homogeneous arrangement of piles and the initially isotropic upper soil layer (before reinforcement), the reinforced soil layer becomes transversely isotropic with the homogenized (effective) characteristics that can be evaluated by different methods:

Voigt homogenization yields the upper bound for effective characteristics (Bensoussan, Lions, Papanicolaou 1978):

$$\mathbf{C}_{effective} = (1-f)\mathbf{C}_{soil} + f\,\mathbf{C}_{piles}, \qquad (1)$$

where $\mathbf{C}_{*}$ are the corresponding elasticity tensors and f is the average volume fraction of piles.

Reuss homogenization:

$$\mathbf{S}_{effective} = (1-f)\mathbf{S}_{soil} + f\,\mathbf{S}_{piles} \qquad (2)$$

yields the lower bound, where $\mathbf{S}_{*}$ are the corresponding compliance tensors. In the case of pile reinforcement these two methods give too broad "fork" and thus, are not reliable.

Much more accurate results give the two-scale asymptotic expansion method (Bensoussan, Lions, Papanicolaou 1978, Sanchez-Palencia 1983):

$$\mathbf{C}_{effective} = (1-f)\mathbf{C}_{soil} + f\,\mathbf{C}_{piles} + \mathbf{K}, \qquad (3)$$

where $\mathbf{K}$ is the corrector that is defined by solving the special boundary value problem for a typical periodical cell. It is interesting to note that taking the corrector $\mathbf{K}$ in Eq. (3) as the null tensor we arrive at Voigt homogenization (1).

Methods for constructing the corrector in the two-scale asymptotic expansion methods are discussed in (Michel, Moulinec, and Suquet 1999, Cecchi & Rizzi 2001).

2 THE MAIN TYPES OF SEISMIC SURFACE WAVES

In this section we proceed to analyzes of the main types of surface waves and conditions for their non-propagation

2.1 *Rayleigh waves*

These waves discovered by Lord Rayleigh (1885) propagate on a plane surface of a halfspace; Fig. 5 and exponentially attenuate with depth.

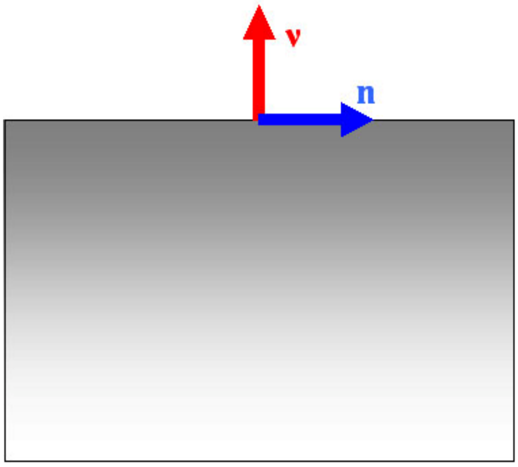

Figure 5. Rayleigh wave in a half-space

The “forbidden” directions of “forbidden” (necessary anisotropic) materials that does not transmit Rayleigh wave along some directions have been intensively searched both experimentally and numerically (Lim & Farnell 1968, 1969, Farnell 1970) until mid seventies when the theorem of existence for Rayleigh waves was rigorously proved (Barnett & Lothe 1973, 1974a,b, Lothe & Barnett 1976 Chadwick & Smith 1977, Chadwick & Jarvis 1979, Chadwick & Ting 1987). This theorem showed that no materials possessing forbidden directions for Rayleigh waves can exist.

Despite proof of the theorem of existence, a small chance for existence of forbidden materials remained. This corresponded to the case of non-semisimple degeneracy of a special matrix associated with the first-order equation of motion (actually, this matrix is the Jacobian for the Hamiltonian formalism). However, it was shown (Kuznetsov 2003) that even at the non-semisimple degeneracy a wave resembling genuine Rayleigh wave can propagate. Thus, for waves propagating on a homogeneous half-space, no forbidden materials or directions can exist.

2.2 *Stoneley waves*

These are waves were introduced by Stoneley (1924), and analyzed by (Sezawa & Kanai 1939, Cagniard 1939, Scholte 1947). Stoneley waves propagate on an interface between two contacting half-spaces, Fig 6.

In contrast to Rayleigh waves, Stoneley waves can propagate only if material constants of the contacting half-spaces satisfy special (very restrictive) conditions of existence. These conditions were studied by Chadwick & Borejko 1994, Sengupta & Nath 2001).

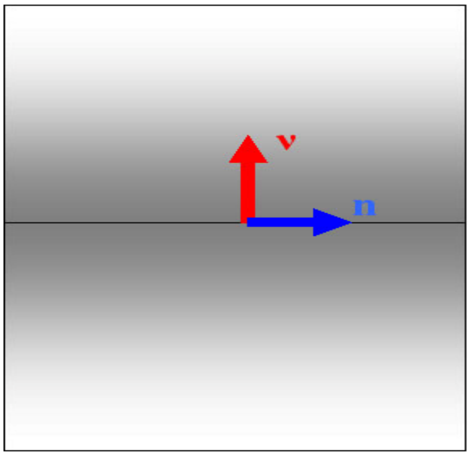

Figure 6. Stoneley wave on the interface

It should be noted that for the arbitrary anisotropy no *closed analytical* relations between material constants of the contacting half-spaces ensuring existence (or nonexistence) of Stoneley waves have been found (2008).

2.3 *Love and SH waves*

Love waves (Love, 1911) are horizontally polarized shear waves that propagate on the interface between an elastic layer contacting with elastic half-space; Fig. 7. At the outer surface of the layer traction-free boundary conditions are generally considered.

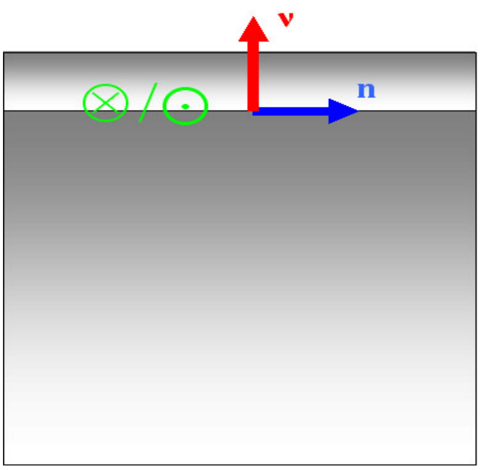

Figure 7. Love wave propagating on the interface

In the case of both *isotropic* layer and half-space the conditions of existence derived by Love are:

$$c_{layer}^{S} < c_{halfspace}^{S}, \tag{4}$$

where c_*^S are the corresponding speeds of the transverse bulk waves. At violating condition (4) no Love wave can propagate. For the case of both anisotropic (monoclinic) layer and half-space the condition of existence is also known (Kuznetsov 2006a).

SH waves resemble Love waves in polarization, but differ in absence of the contacting half-space. At the outer surfaces of the layered plate different boundary conditions can be formulated (Kuznetsov 2006b). In contrast to Love waves, the SH waves exist at any combination of elastic properties of the contacting layers.

Besides Love and SH waves a combination of them can also be considered. This corresponds to a horizontally polarized wave propagating in a layered system consisting of multiple layers contacting with a half-space. Analysis of conditions of propagation for such a system can be done by applying either transfer matrix method (Thomson 1950, Haskell 1953), known also as the Thomson-Haskell method due to its originators; or the global matrix method developed by Knopoff (1964).

At present (2008) no closed analytical conditions of existence for the combined Love and SH waves propagating in anisotropic multilayered systems are known; however, these conditions can be obtained numerically by applying transfer or global matrix methods; see (Kuznetsov 2006a, b).

Different observations show that genuine Love and the combined Love-SH waves along with Rayleigh and Rayleigh-Lamb waves play the most important role in transforming seismic energy at earthquakes (e.g. Agnew 2002, Braitenberg & Zadro 2007). But, as we have seen, there is a relatively simple (at least from theoretical point of view) method for stopping Love and the combined Love and SH waves by modifying the outer layer in such a way that conditions of existence are violated.

2.4 *Lamb and Rayleigh-Lamb waves*

Lamb waves (Lamb, 1917) are waves propagating in a homogeneous plate and (if a plate is isotropic) polarized in the saggital plane, similarly to polarization of the genuine Rayleigh waves. It is known (Lin & Keer 1992, Ting 1996) that Lamb waves can propagate at any anisotropy of the layer and at traction-free, clamped, or mixed boundary conditions at the outer surfaces of the plate. The same result can be extrapolated to a layered plate containing multiple anisotropic homogeneous layers in a contact (Ting 2002). Thus, for Lamb waves no forbidden materials exist.

More interesting from seismological point of view are Rayleigh-Lamb waves that are also polarized in the saggital plane and propagate in a system of layers contacting with a halfspace. Such a layered structure resembles one where Love or the combined Love-SH waves propagate, but Rayleigh-Lamb waves obviously differ from Love waves in polarization.

Theoretical research in developing longitudinal seismic barriers for Rayleigh-Lamb waves are currently focused on (i) finding conditions that can be imposed on the physical properties the outer layer to prevent Rayleigh-Lamb waves from propagation; and (ii) setting up an optimization problem on minimizing amplitudes of deflections or accelerations of the traveling Rayleigh-Lamb wave by varying physical properties of the outer layer.

Mathematically the optimization problem for minimizing amplitudes of deflections can be written as finding minimum of the following target function:

$$\min_{\mathbf{C}_1,\rho_1}\left(\max_{\omega\in\Omega}\ \max_{-h_1/2\le x\le h_1/2}\left[s(\omega)u_1(x,\omega)\right]\right), \tag{5}$$

where $\mathbf{C}_1,\rho_1$, and h_1 are the elasticity tensor, density, and depth of the first (outer) layer correspondingly, ω is the angular frequency, Ω is a set the interesting frequencies, $s(\omega)$ is the spectral density, $x=\mathbf{v}\cdot\mathbf{x}$ is a coordinate along depth of the layer, and u_1 is the amplitude of deflections in this layer. This problem resembles one that is usually solved at finding optimal parameters for shock absorbers (Den Hartog 1985, Balandin et al. 2000, 2008).

2.5 *Solitonlike waves*

Love, SH, and Rayleigh-Lamb waves admit propagation of their peculiar branches having infinite wavelength. These peculiar waves can arise at blast underground impacts and resemble solitons in hydrodynamics.

Recent studies of the SH solitonlike waves (Djeran-Maigre & Kuznetsov 2008) revealed that conditions for stopping their propagation are essentially the same as for the genuine SH waves.

3 BASIC STEPS IN THE WAVE BARRIERS DEVELOPMENTS

3.1 *Simulation of surface acoustic waves propagation*

This stage includes (i) analyses of capabilities of the main computer codes for dynamical non-stationary modeling of SAW propagation, including Rayleigh, Lamb, Rayleigh-Lamb, Love, and SH waves; and (ii) interaction of the initial bulk waves arising at the source of perturbation with plane boundaries and forming SAW.

3.2 *Modeling interaction of the SAW with piles*

Again, as it was in the preceding stage FEM analysis will be used to model (i) interaction of the SAW with a single pile; and (ii) scattering SAW by multiple piles (pile field).

3.3 *Performing two-scale asymptotic analysis in finding homogenized properties of the pile field*

At this stage (i) the so-called cell problem will be solved by applying either periodic boundary integral equation technique (Kaptsov, Kuznetsov 1998), or by use of the FEM codes; (ii) the overall homogenized properties of the pile field will be obtained by the two-scale asymptotic analysis (Sanchez-Palencia 1983).

3.4 *Finding properties of the surface layer that prevents particular SAW to move into protected zone, or minimizing the amplitude of vibrations in the protected region*

This stage is mainly based on (i) different analytical methods (Kuznetsov 2002, 2003, 2006a, b) describing physical properties of the surface layer needed to prevent a particular SAW to propagate into the protected zone, or (ii) finding conditions imposed on the surface layer, at which the amplitude of the SAW decreases in the protected zone.

3.5 *Analyzing interaction of the SAW with the modified surface layer*

This the final stage for the project. Actually, by applying FEM analysis, it should demonstrate that the particular SAW does not propagate inside the region surrounded by the barrier made by the pile field that desirably modifies the surface layer.

4 SUMMARY

Thus, the suggested principle for creating longitudinal barriers against surface seismic waves can either (i) prohibit these waves from penetrating into the protected region, or (ii) decrease the amplitude of deflections in the outer layer by choosing its physical properties that ensure minimum of the target function (5).

It should also be noted that in some circumstances the proposed method can be the only tool to protect building and structures, here we mainly mean soils subjected to liquefaction. Figure 8 demonstrates an overturn of the building as a result of the earthquake that caused soil liquefaction.

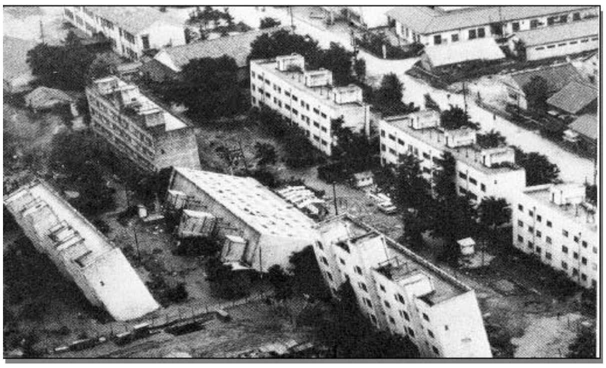

Figure 8. Overturn of the building due to liquefaction (Nnigata, 1964) By courtesy of FEMA

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