THE USE OF GUIDED WAVES TO CHARACTERIZE WATER SATURATED LAYERS

DEVELOPMENT OF THE THEORETICAL METHOD

proposal prepared by

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Executive Summary

The main target of the Project: The proposed research is an integrated analytical and numerical approach for developing a mathematical model and the corresponding numerical algorithms for determining water saturated depositions beneath the Earth surface. The method under development, is focusing on creating a robust, yet nondestructive tool for monitoring condition of the undersurface layers subjected to possible degradation of their physical properties due to the deep penetration of soil waters. The latter problem appears to be one of the extremely challenging for the whole civil engineering, apparently since its origin. In this respect we mention only one, but famous structure that has been suffering from the water saturated soils for several centuries, that is Pisa Tower.

Other objectives: Along with creating a tool for determining water saturated layers and monitoring their state, the developing theoretical method can also be applied to analyzing presence of the undersurface anomalies of different origin, as oil and gas fields, underground cavities, etc. Beside that, the discussed method, as being non-evasive, can find applications in different kind of the human activity, where non-destructive methods of analyses are needed, from microelectronics to auto-, marine-, and aviation-technology.

Mathematical model: Principles of constructing the mathematical model are depicted on Fig.1.

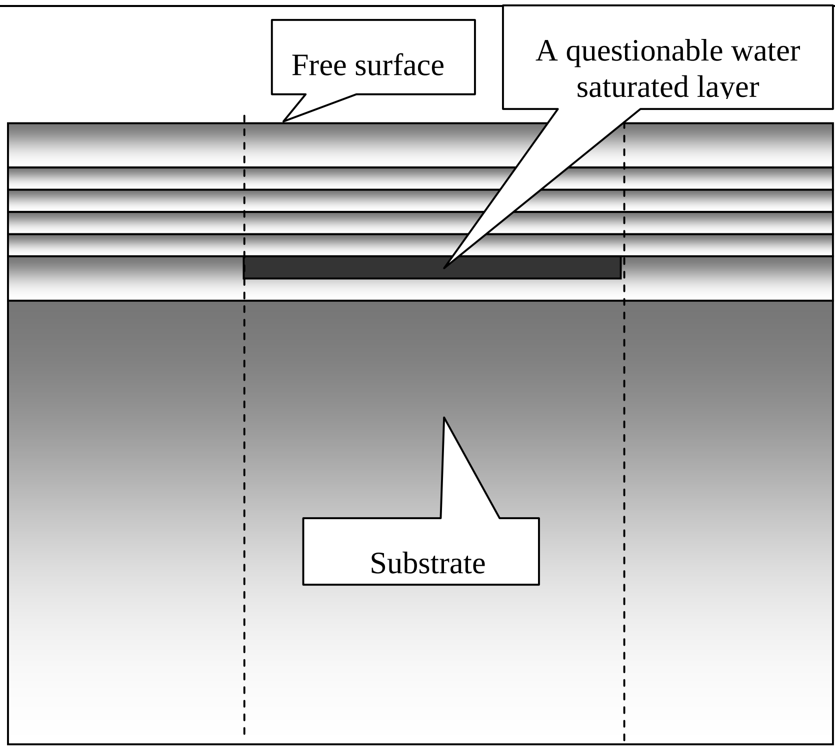

Figure 1. A mathematical model for the water saturated layer

Appearing an additional water saturated layer, whose physical properties are different to the other layers in respect of the almost vanishing shear resistance, dramatically changes structure of the dispersion properties of the horizontally polarized surface waves. In the regions, where there is no water saturated layer, the corresponding surface waves are of Love type (outside dashed lines on Fig.1), while in the region containing such a layer, the corresponding surface waves mainly belong to the SH type.

Since the dispersion properties of Love and SH waves differ considerably, there appears a possibility to identify water saturated depositions by analyzing the corresponding dispersion data. However, the real geological structure of can be much more complicated consisting of layers with continuously varying depth and variable slope; see Fig. 2.

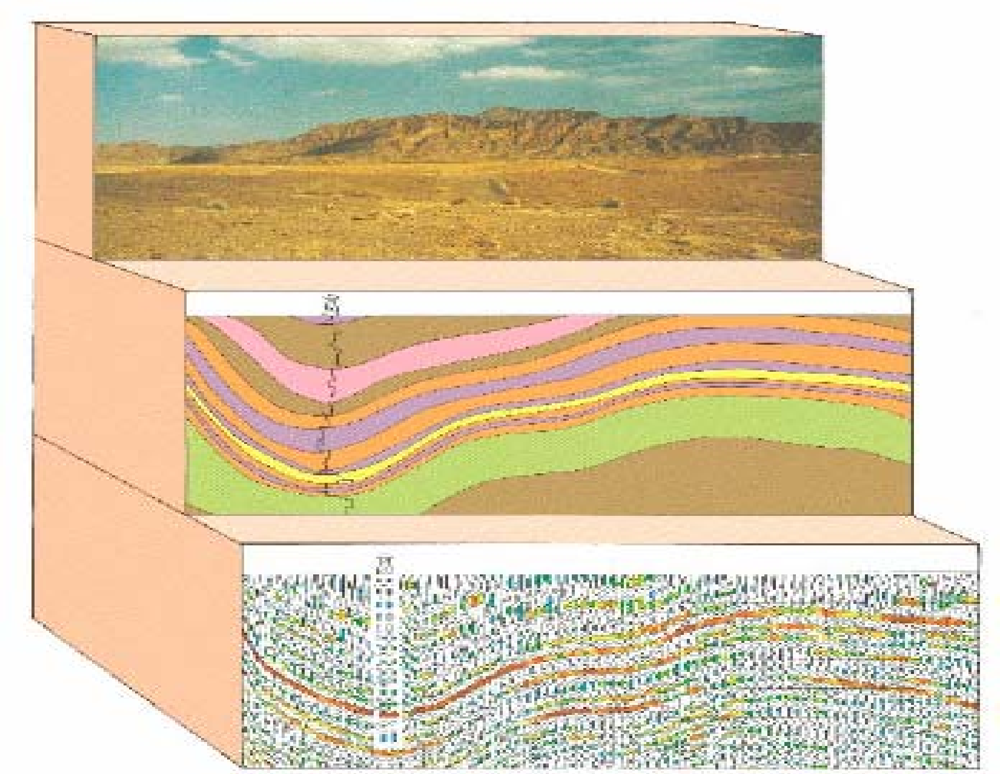

Figure 2. Layered structure of a geological profile

At present, the direct sonic scanning (Fig. 2) is mainly used for obtaining information on a possible geological profile. While this method is simple in realization, it has the main disadvantage due to its low reliability, because of (i) huge scattering of the bulk P-waves used for such measurements; (ii) multiple reflections of these waves from the undesired objects; (iii) principle inability to analyze deep layers (typically this method is used at a depth not exceeding 50 meters, while at higher depth the results are becoming unreliable); and (iv) poor information obtained due to their non-dispersion nature.

The basic idea of the proposed research project is to develop an alternative acoustical method based on incinerating surface waves (primarily SH and Love waves as they are very sensitive to variation of shear properties of the sedimentary layers), instead of using bulk waves. Experimental and theoretical observations (see our brief review of the current publications below), show that surface waves compared to the bulk waves have (i) much lower scattering; (ii) their penetrating ability is almost unrestricted, and depends only upon their wavelength; (iii) SH and Love waves are highly dispersive, so much more information can be obtained by analyzing the corresponding dispersion curves; the latter results in (iv) much more reliable data that can be obtained by using the surface acoustic waves.

Methods of analyses: The main theoretical method to be developed, will be based on analyzing variation of speed and dispersion of the surface waves (SH, Lamb, and Love waves) propagating in the multilayered media, containing various layers. The method represents a combined approach utilizing the special six-dimensional complex formalism and a modified Transfer Matrix method. Both the six-dimensional formalism and the modified Transfer Matrix method have already been applied to solving different problems of wave dynamics (Kuznetsov, 1998-2005; Djeran-Maigre, Kuznetsov, 2004).

Solution of the inverse problem on identifying physical and geometrical properties of a questionable layer will be obtained by applying genetic algorithms based on combination of a regularization technique for solving the ill-posed problems and the high precision numerical packages (Bailey, 2002) allowing one to perform the main computations with numbers having large mantissas, typically 100 – 1000 decimal digits (Kuznetsov, 2004).

Anticipated results: In the course of the proposed research effort, the following results should be achieved:

1. Theoretical methods will be developed, capable of analyzing dispersion relations of guided SH, Lamb, and Love waves propagating in multilayerd media composed of large number of anisotropic layers having arbitrary elastic anisotropy. The developing methods are based on the recent works by members of the team Kuznetsov (1998-2005), Djeran-Maigre and Kuznetsov (2005) devoted to analyzing propagation of Rayleigh, Lamb, SH, and Love surface waves in stratified media.
2. A series of numerical algorithms, ready for the commercial usage, will be worked out, utilizing high precision arithmetic and capable of analyzing wave propagation in the stratified media containing very large number of layers of the different structure (typically > 30), that is needed for an adequate modeling of the real situations in soil and geomechanics. At present, the main part of this stage has been realized in a working computer code.
3. A regularization technique should be developed for obtaining numerically stable solutions to the inverse problems on identifying physical properties and structure of the internal layers by analyzing the corresponding dispersion relations. This stage is currently being analyzed, and it is partly implemented in numerical algorithms for obtaining solutions related to the non-destructive testing of some multilayered plates used in microelectronics industry.

4. A lecture course for the university students will be developed, tentatively entitled “Methods of wave dynamics and NDT in geo-engineering”.
5. A monograph on methods of wave dynamics in geo-engineering will be finished and submitted for publication.

Regarding its high importance to the problem of defining and monitoring water saturated layers, a continuation of the research plan is foreseen by the following experimental part Part II. Model and full scale “in situ” experiments.

State of the art: The problem of predicting water or oilfields or water saturated layers by analyzing surface wave propagation has roots going back to the middle of the last century, when the basic techniques and the acoustical methodology were established, (Brekhovskih, Hankell, Knopoff, Mal, Maradudin, Thomson, and Victorov). These works gave the necessary basis for developing the corresponding experimental technique. However, at that time almost all theoretical works concerning surface wave propagation, dealt with isotropic materials. The appropriate theory for Rayleigh waves, propagating in anisotropic media with an arbitrary anisotropy, was mainly developed towards the end of the 80^{th} mainly by Barnett and Lothe (1973-1987), Chadwick et al. (1977, 79), and Ting (1983-1989). Other types of surface waves have been theoretically studied within recent years mainly by Alshits (1983-2005), Kuznetsov (1998-2006), Lowe (1995, 2002), Ting (1995-2003).

Despite resolving several basic problems on Rayleigh, Lamb, SH, and Love wave propagation in *homogeneous* anisotropic media, the major technical difficulties known as the large *fd*-problem (*f* is the frequency of a wave and *d* is the depth of the layer), prevent from developing a technique for analyzing layered media with large number of layers with the considerable difference in depth. Presumably, five anisotropic layers composing a layered plate, is a natural limit for the traditional approaches in Lamb wave analyses; see Achenbach (1998), Yang and Kundu (1998), Mallah, Philippe, and Khater (1999), Simons, Zielhuis, and van der Hilst (1999), Hisada, Yamamoto, and Tani (2001), Doxbeck et al. (2002), Karchevskij (2005), Safaeinili et al. (1995), Potel et al., (1999) where the latter two are devoted to media with periodic layers. At the same time, to be reliable and adequate, the corresponding mathematical model should be capable of analyzing stratified media having 20 – 30 anisotropic layers, that suits typical geological observations.

Such a theory is currently being under development by the joint team members. This theory is based on (i) a newly developed variant of the six-dimensional formalism, (ii) a modified transfer matrix method, (iii) the variant of Tykhonov’s regularization technique, and (iv) a multi precision computer package developed by Dr. Bailey. Altogether, these four amendments and modifications of the existing approaches along with the already obtained theoretical and numerical results, allows members of the joint team to be aware in a successful completion of the proposed Project.

Basic stages. The basic stages for the regarded project are assumed to be the following ones:

The first year of the project will be mainly devoted to completing theoretical and numerical analyzes on propagation of Love and SH waves in stratified anisotropic media;

namely, (i) the existing approach covering media with the monoclinic symmetry will be extrapolated to the layered media with arbitrary elastic anisotropy, (ii) the corresponding numerical algorithms will be redeveloped; (iii) at least two scientific paper on SH, Love, and Lamb waves propagation in layered media with arbitrary anisotropy will be finished and submitted for publication; (iv) starting work on manuscript on surface waves; (v) organizing a workshop on surface waves in NDT of multilayered structures.

The second year will be devoted to (i) completing the numerical algorithm on propagation waves in layered media with arbitrary elastic anisotropy; (ii) obtaining the corresponding numerical results; (iii) starting work on developing Tykhonov's regularization technique for solving the inverse problem on identifying properties of the internal layers by analyzing the dispersion curves; (iv) continuation of the work on the manuscript on surface waves; (v) starting work on developing a lecture course on the NDT in geo-engineering.

The third year of the project will mainly be devoted to (i) obtaining and analyzing numerical data on propagation of the SH, Love, and Lamb waves in stratified anisotropic media; (ii) completing the numerically stable algorithm on analyzing properties of the internal layers; (iii) submitting at least two scientific papers on the surface wave analyses for publication; (iv) finishing work on the manuscript on surface waves; and (v) continuation of the work on developing a lecture course on the NDT in geo-engineering; (vi) starting a literature survey for the new the proposal (Part II).

The fourth year will be devoted to (i) combining all the developed theoretical approaches on the surface waves in stratified media and presenting a unified approach on Lamb, Love, and SH wave propagation in stratified media with arbitrary elastic anisotropy (ii) combining all the numerical algorithms in the corresponding numerical packages; (iii) disseminating the developed packages to the universities and geophysical institutions; (iv) preparing at least two scientific papers on the unified approach for analyzing surface waves in multilayered anisotropic media; and (v) completing the lecture course on NDT in geo-engineering; (vi) developing and submitting a new proposal entitled: "The use of guided waves to characterize water saturated layers. Part II. Development of the Experimental Methods".

Possible other applications: While principally oriented to addressing the characterization and monitoring water saturated depositions, the proposed research effort can also have applications in several different areas of science and technology, where nondestructive evaluation of material properties of a stratified medium is needed: seismology, acoustical microscopy, composite manufacturing, biomechanics, etc. Of particular importance in this regard, is the ability to characterize physical and geometrical properties of very thin films as they are deposited. Moreover, the methods under development in this research program are well suited for the real-time on-line process control applications, as these methods are nondestructive, non-contact and minimally invasive.

The successful completion of this research effort will promote a better understanding nature of the surface waves and giving methods for predicting properties and structure of the internal layers inaccessible by the direct measurements.

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