

# Principles and Methods of Seismic protection

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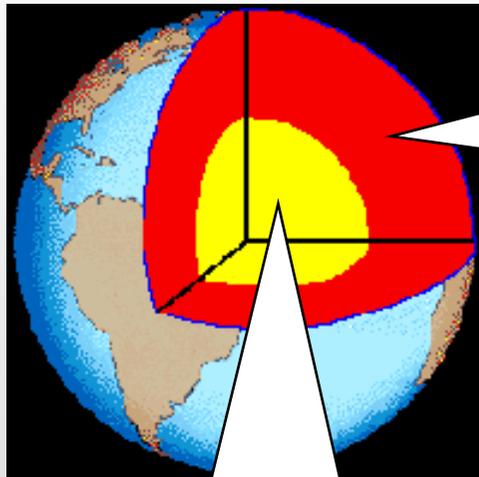
# Part I. General considerations

# 1. Introduction

## 1.1. Position of acoustic methods in rock mechanics

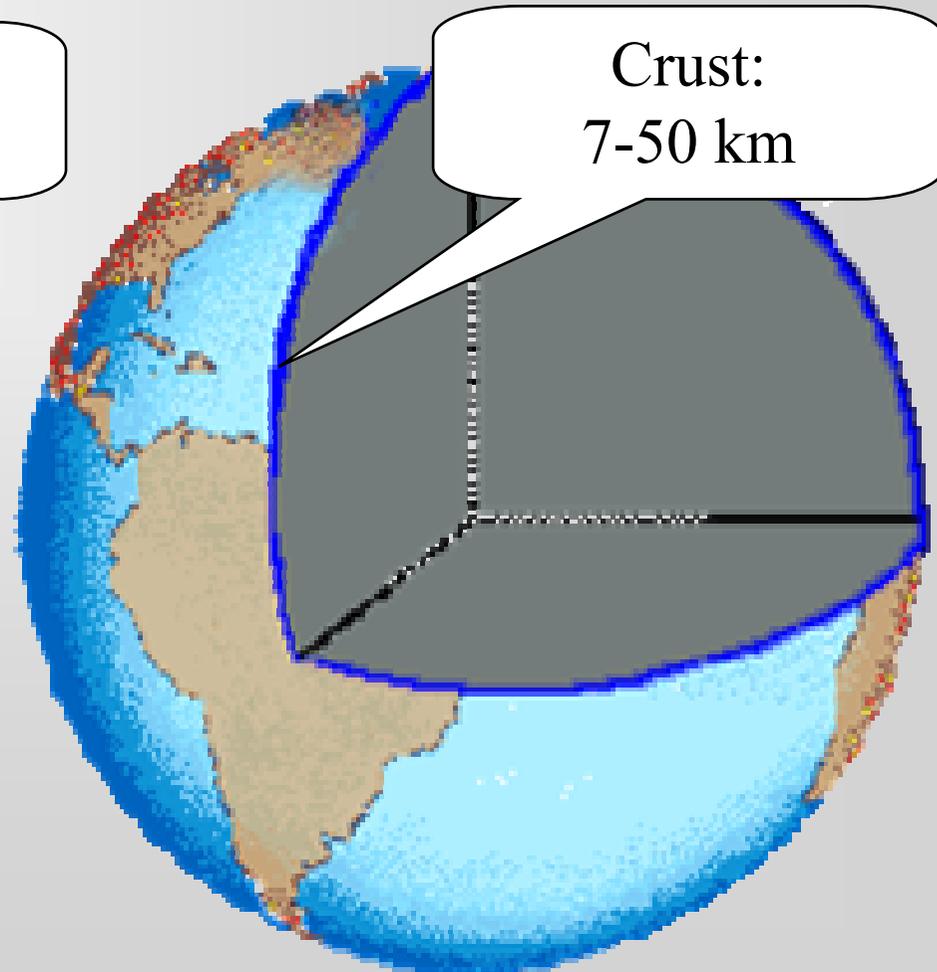
### 1.1.1. Earth's structure

#### 1.1.1.1. Earth's schematic structure



Mantle:  
2885 km

Earth's Core: inner  
core  $\varnothing$  1216 km +  
outer core 2270 km



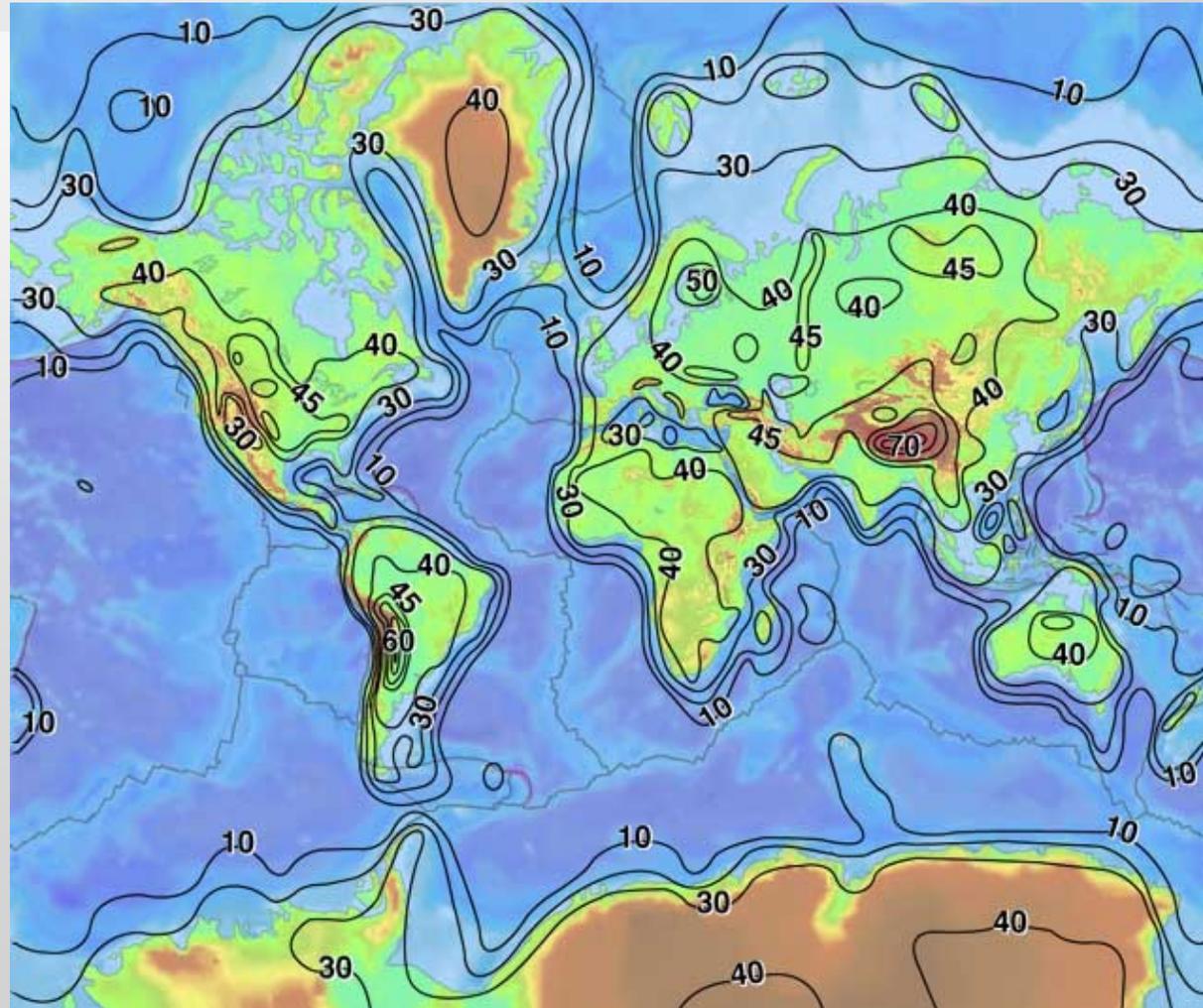
Crust:  
7-50 km

# 1. Introduction

## 1.1. Position of acoustic methods in rock mechanics

### 1.1.1. Earth's structure

#### 1.1.1.2. Depth of Earth's crust



# 1. Introduction

## 1.2. Some experimental data concerning propagation of acoustic waves in rocks and soils

### 1.2.1. Basic definitions

A wave propagating in a given material is called **subsonic** (**supersonic**), if its speed is below (greater) than the corresponding speed of the longitudinal wave propagating in the same direction

A wave is called **hyposonic** (**ultrasonic**), if its oscillations in time are below (greater) than the audible frequencies: 20 – 20 000 Hz

# 1. Introduction

## 1.2. Some experimental data concerning propagation of acoustic waves in rocks and soils

### 1.2.2. Frequency range

Wave nature	Frequency range
Seismic waves of natural origin	0.001 - 50Hz  Most dangerous: 1-30Hz
Waves of artificial nature	10 - 120Hz  Most dangerous: 10-70Hz

# 1. Introduction

1.2. Some experimental data concerning propagation of acoustic waves in rocks and soils

1.2.3. Speed range for *longitudinal waves*

Material	Speed m/sec
air	330 - 360
Bullet in the air	~800
soil	200 - 800
sand	100 - 1000
water	1430 - 1590
slate (shale)	2000 - 5000
limestone	3000 - 6000
granite	4500 - 6500

# 1. Introduction

## 1.2. Some experimental data concerning propagation of acoustic waves in rocks and soils

### 1.2.4. Wavelength ranges

#### 1.2.4.1. Wavelength range for seismic *longitudinal* waves

Wavelengths for seismic longitudinal waves from 1 to 50 Hz

$$l = c / \omega,$$

where

$l$  is wavelength

$c$  is speed

$\omega$  is frequency

Material	Wavelength m
soil	40 - 800
sand	2 - 1000
water	29 - 1590
slate (shale)	40 - 5000
limestone	60 - 6000
granite	90 - 6500

# 1. Introduction

## 1.2. Some experimental data concerning propagation of acoustic waves in rocks and soils

### 1.2.4. Wavelength ranges

#### 1.2.4.2. Remark for wavelength of seismic waves

$$l = c / \omega,$$

where

$l$  is wavelength

$c$  is speed

$\omega$  is frequency

As was pointed out previously there can be seismic waves propagating with very low frequencies (0.01-1Hz), for such waves the corresponding wavelength can be sufficiently larger, than pointed in the previous table. Thus, for granites the wavelength of longitudinal waves can have up to 650 km.

# 1. Introduction

1.2. Some experimental data concerning propagation of acoustic waves in rocks and soils

1.2.4. Wavelength ranges

1.2.4.3. Wavelength range for artificial longitudinal waves

Wavelengths for artificial longitudinal waves from 10 to 70 Hz

$$l = c / \omega,$$

where

$l$  is wavelength

$c$  is speed

$\omega$  is frequency

Material	Wavelength m
soil	28 - 80
sand	1 - 100
water	20 - 159
slate (shale)	28 - 500
limestone	42 - 600
granite	63 - 650

# 1. Introduction

## 1.3. Seismic waves scales

### 1.3.1. Richter magnitude scale

Charles Richter (1935) arbitrarily chose a magnitude 0 event to be an earthquake that would show a maximum combined horizontal displacement of 1 micrometer on a seismogram recorded using a Wood-Anderson torsion:

$$M_L = \log_{10} A(mm) + (\text{Distance correction factor})$$

#### Remark

According to Richter scale, earthquakes of magnitude

<3	are not felt	(frequency is ~1000 per day)
5 - 6	can cause damage	(~800 per year)
7 - 8	serious damage	( ~18 per year)
8 - 9	severe damage	( ~1 per year)
>9	extreme damage	( ~1 per 20 years)

# 1. Introduction

## 1.3. Seismic waves scales

### 1.3.2. Mercalli *intensity* scale

The *Modified Mercalli Intensity Scale* (originated to seismologist Giuseppe Mercalli, 1902) is commonly used by assigning numbers **I – XII** according to severity of the earthquake effects, so there may be many the modified Mercalli intensity values for each earthquake, depending upon distance of the epicenter.

#### Remark

According to the Mercalli modified intensity scale:

- I. People do not feel any Earth movement.
- II. A few people might notice movement.
- III. Many people indoors feel movement.
- IV. Most people indoors feel movement.

.....

- XII. Almost everything is destroyed.

# 1. Introduction

## 1.3. Seismic waves scales

### 1.3.3. The greatest earthquake

According to Richter scale, the greatest recorded earthquake occurred on 22d May, 1960 in Chili (The Great Chilean Earthquake or Valdivia Earthquake).

This earthquake was measured 9.5 by Richter scale.

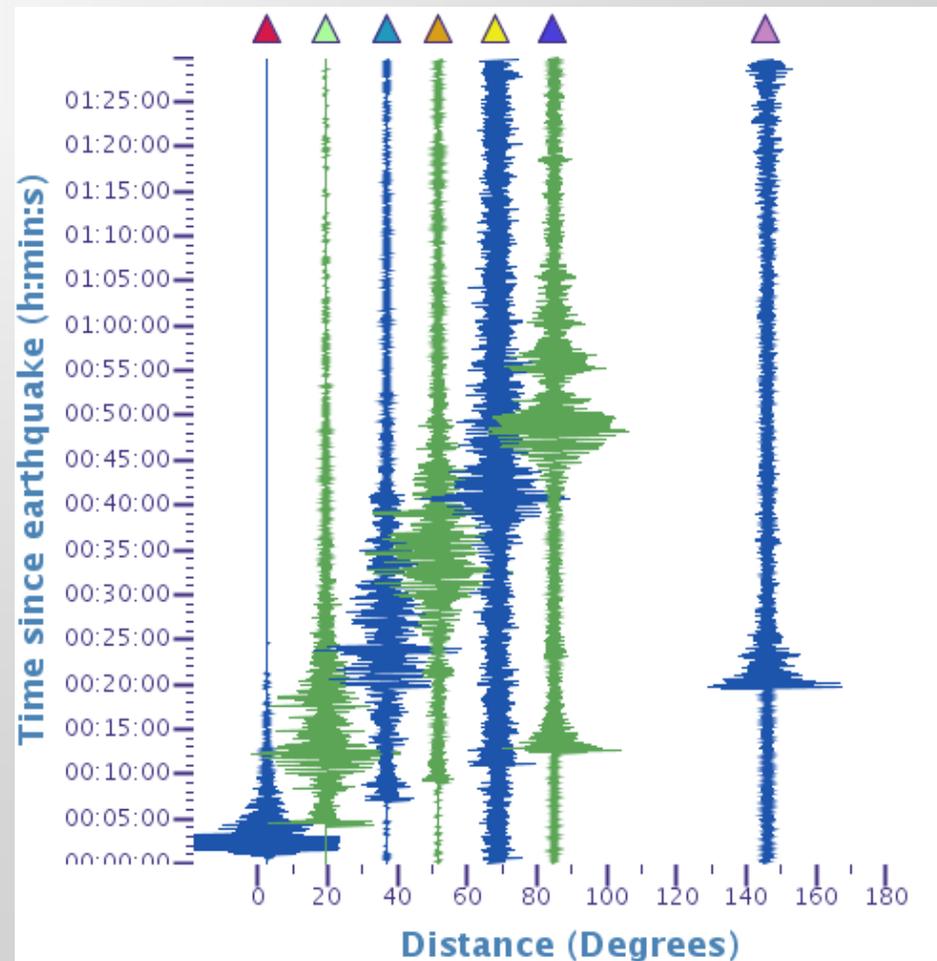
#### **Remark**

The earthquake caused localized tsunami that hit the Chilean coast severely, with waves up to 25 meters high. The main tsunami ran through the Pacific Ocean and hit Hawaii, where waves as high as 10.7 meters high were recoded

# 1. Introduction

## 1.4. Typical seismograms

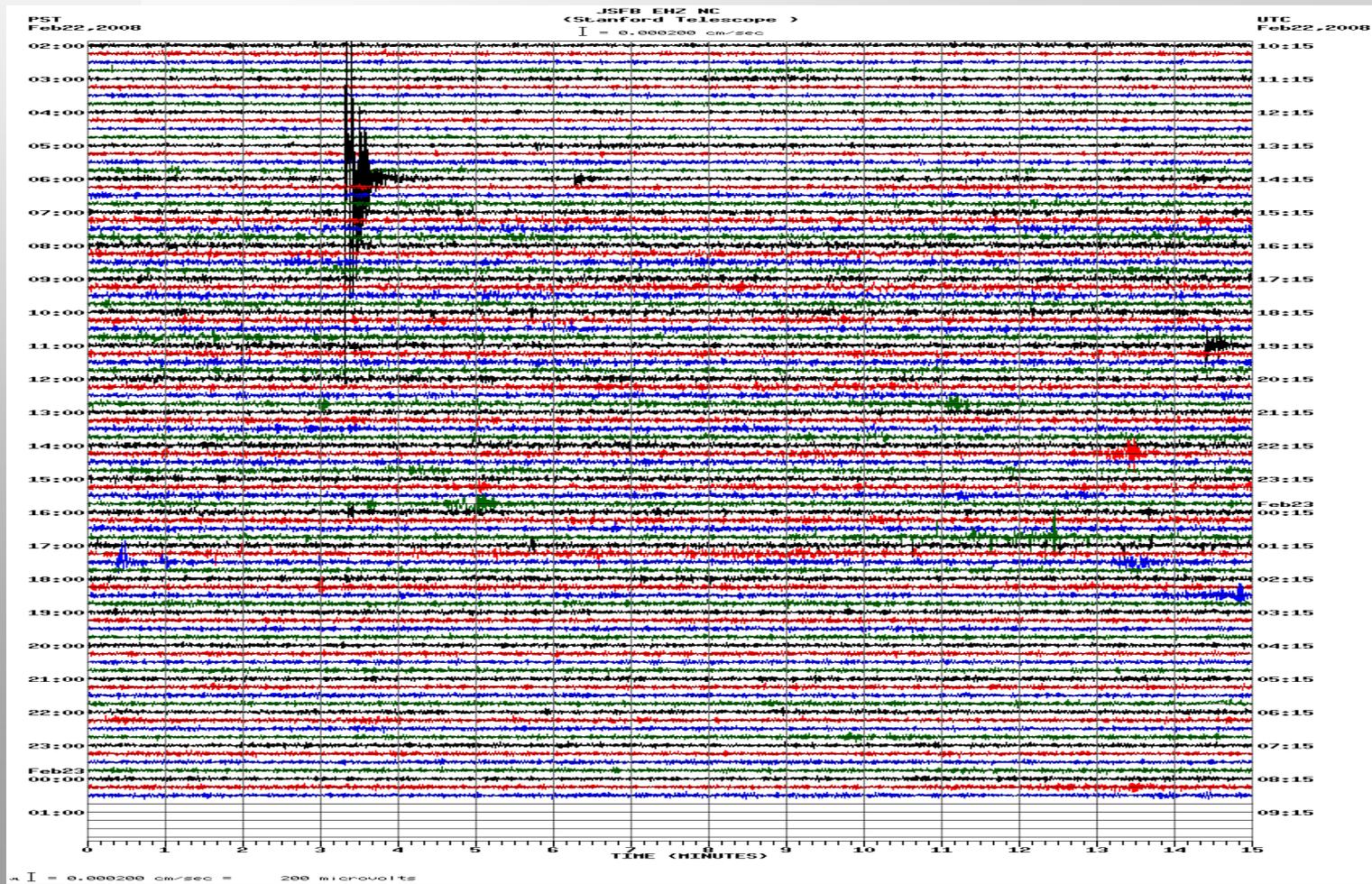
### 1.4.1. Earthquake 6.7MO in Southern Greece 14/02/2008



# 1. Introduction

## 1.4. Typical seismograms

### 1.4.1. North California Seismic Station, Feb 2008



# 1. Introduction

## 1.5. Consequences of the Earthquakes (Kobe, 1995)

### 1.5.1. Overview

At 5:46 in the morning, a magnitude 6.9 (M<sub>w</sub>) earthquake struck Kobe in Japan. About 5,500 people died and 35,000 were injured.

Nearly 180,000 buildings were damaged or destroyed, leaving more than 300,000 people homeless that night

The Earthquake Engineering Research Center  
University of Bristle

Damage to buildings: Fully collapsed 67,421 structures  
Partially collapsed 55,145 structures

The Great Hanshin-Awaji Earthquake Statistics and Restoration Progress January 1, 2008  
<http://www.city.kobe.jp/cityoffice/06/013/report/january.2008.pdf>

# 1. Introduction

## 1.5. Consequences of the Earthquakes (Kobe, 1995)

### 1.5.2. Local damage



The Earthquake Engineering Research Center  
University of Bristle

City of Kobe. City council

# 1. Introduction

## 1.5. Consequences of the Earthquakes (Niigata, 1964, Kobe, 1995)

### 1.5.3. Liquefaction

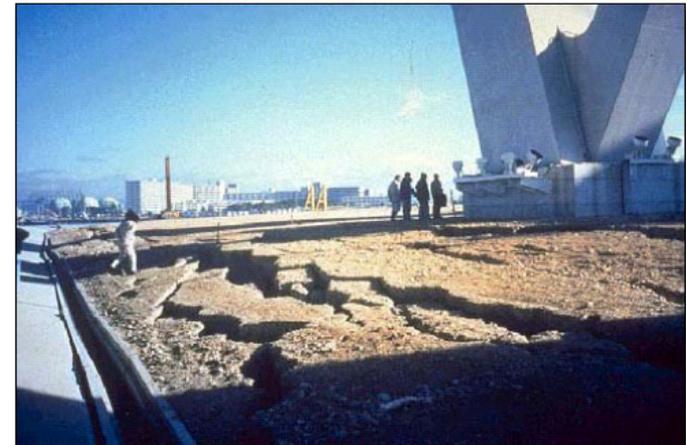
#### Liquefaction Damage, Niigata, Japan, 1964



Instructional Material Complementing FEMA 451, Design Examples

Earthquake Mechanics 2 - 22

#### Liquefaction and Lateral Spreading, 1993 Earthquake in Kobe, Japan



Instructional Material Complementing FEMA 451, Design Examples

Earthquake Mechanics 2 - 23

# 1. Introduction

## 1.5. Consequences of the Earthquakes (Kobe, 1995)

### 1.5.4. Local ground faults



From a [report](#) by J.-P. Bardet  
at [USC](#) and others at Gifu Univ.;

# 1. Introduction

## 1.5. Consequences of the Earthquakes (Kobe, 1995)

### 1.5.5. Different (local) damage intensity



From City Kobe City Office

<http://www.city.kobe.jp/cityoffice/15/020/quake/teiten/images/>



# 1. Introduction

## 1.5. Consequences of the Earthquakes (Kobe, 1995)

### 1.5.6. Conclusions

The wave nature of seismic activity should be taken into account, when seismic protection is developed

This is implemented in EC8 (EU) and BCJ (Japan) for S-waves traveling in layered soils:

E.M.Marino et al, Engineering Structures 27 (2005) 827–840

The existing methods for seismic protection need in revision

The newly developed methods and principles of seismic protection should be foreseen

## Part II. Surface acoustic waves

# 4. Theoretical methods in acoustical studies

## 4.3. Bulk wave propagation in anisotropic elastic media

### 4.3.1. Introduction

#### 4.3.1.1. Basic definitions

##### Definitions

A wave is a periodic or quasi periodic movement in time and space.

Wave front is the geometrical set of points vibrating with the same phase.

##### Types of waves according to the wave front

Spherical waves

Cylindrical waves

Waves with a plane wave front

# 4. Theoretical methods in acoustical studies

## 4.3. Bulk wave propagation in anisotropic elastic media

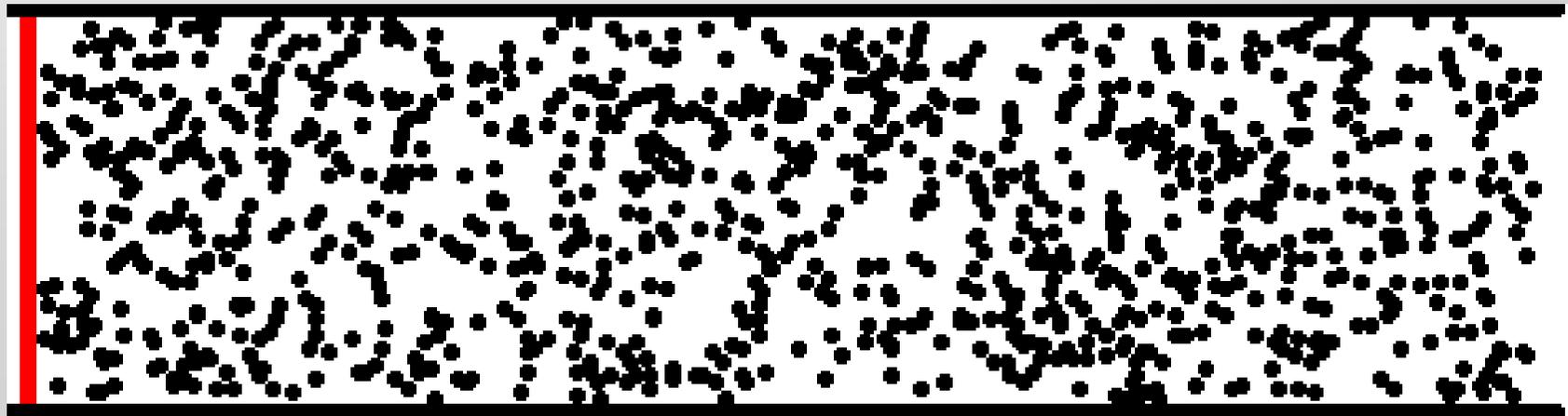
### 4.3.1. Introduction

#### 4.3.1.2. Remark on no mass transfer

##### Remark

At wave motion no mass transfer occurs.

This is applied to all the (linear) theories of acoustic waves.



## 4. Theoretical methods in acoustical studies

### 4.3. Bulk wave propagation in anisotropic elastic media

#### 4.3.2. The main equations for bulk waves

##### 4.3.2.1. Representation for a wave with the plane wave front

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{m} e^{ir(\mathbf{n} \cdot \mathbf{x} - ct)}$$

Where

$\mathbf{u}$  is a displacement field

$\mathbf{x}$  is a space variable

$t$  is time

$\mathbf{m}$  is the amplitude (polarization) of the wave

$r$  is the wave number ( $r = 2\pi/l$ , or  $r = \omega/c$ )

$\mathbf{n}$  is direction of propagation ( $\mathbf{n}$  is the unit vector)

$c$  is the phase speed

## 4. Theoretical methods in acoustical studies

### 4.3. Bulk wave propagation in anisotropic elastic media

#### 4.3.2. The main equations for bulk waves

##### 4.3.2.2. Acoustic tensor

Substituting representation for the plane wave front into equation of motion yields

$$ir \left( \mathbf{n} \cdot \mathbf{C} \cdot \mathbf{n} - \rho c^2 \mathbf{I} \right) \cdot \mathbf{m} = 0$$

#### Definition for the acoustic tensor

$$\mathbf{A}(\mathbf{n}) \equiv \mathbf{n} \cdot \mathbf{C} \cdot \mathbf{n}$$

#### Remark

The acoustic tensor can be constructed for any direction  $\mathbf{n}$ , and it is symmetric and positive definite for any kind of elastic anisotropy

# 4. Theoretical methods in acoustical studies

## 4.3. Bulk wave propagation in anisotropic elastic media

### 4.3.2. The main equations for bulk waves

#### 4.3.2.3. Christoffel equations

$$\left(\mathbf{A}(\mathbf{n}) - \rho c^2 \mathbf{I}\right) \cdot \mathbf{m} = 0 \quad \longrightarrow \quad \det\left(\mathbf{A}(\mathbf{n}) - \rho c^2 \mathbf{I}\right) = 0$$

$$\downarrow$$
$$\mathbf{Q}^t \cdot \left(\mathbf{D}_{\mathbf{A}(\mathbf{n})} - \rho c^2 \mathbf{I}\right) \cdot \mathbf{Q} = 0$$

Remark

$$\mathbf{D}_{\mathbf{A}(\mathbf{n})} = \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{pmatrix}$$



$$c_k = \sqrt{\frac{\lambda_k}{\rho}}, \quad k = 1, 2, 3$$

## 4. Theoretical methods in acoustical studies

### 4.3. Bulk wave propagation in anisotropic elastic media

#### 4.3.2. The main equations for bulk waves

##### 4.3.2.4. Polarization

$$\mathbf{A}(\mathbf{n}) = \lambda_1 \mathbf{m}_1 \otimes \mathbf{m}_1 + \lambda_2 \mathbf{m}_2 \otimes \mathbf{m}_2 + \lambda_3 \mathbf{m}_3 \otimes \mathbf{m}_3$$

where

$\mathbf{m}_k, k = 1, 2, 3$  Are the mutually orthogonal and normal eigenvectors of the acoustical tensor

**Corollary**

Polarization vectors corresponding to different eigenvalues of the acoustic tensor are just its eigenvectors

## 4. Theoretical methods in acoustical studies

### 4.3. Bulk wave propagation in anisotropic elastic media

#### 4.3.2. The main equations for bulk waves

##### 4.3.2.5. Classification of bulk waves according to polarization

#### Definitions

A wave is called **longitudinal** (or **P-wave**), if polarization  $\mathbf{m}$  coincides with the direction of propagation  $\mathbf{n}$

A wave is called **transverse** (or **S-wave**), if polarization  $\mathbf{m}$  is orthogonal to the direction of propagation  $\mathbf{n}$

---

A wave is called **quasi longitudinal**, if the scalar product  $\mathbf{m} \cdot \mathbf{n} > 0$ .

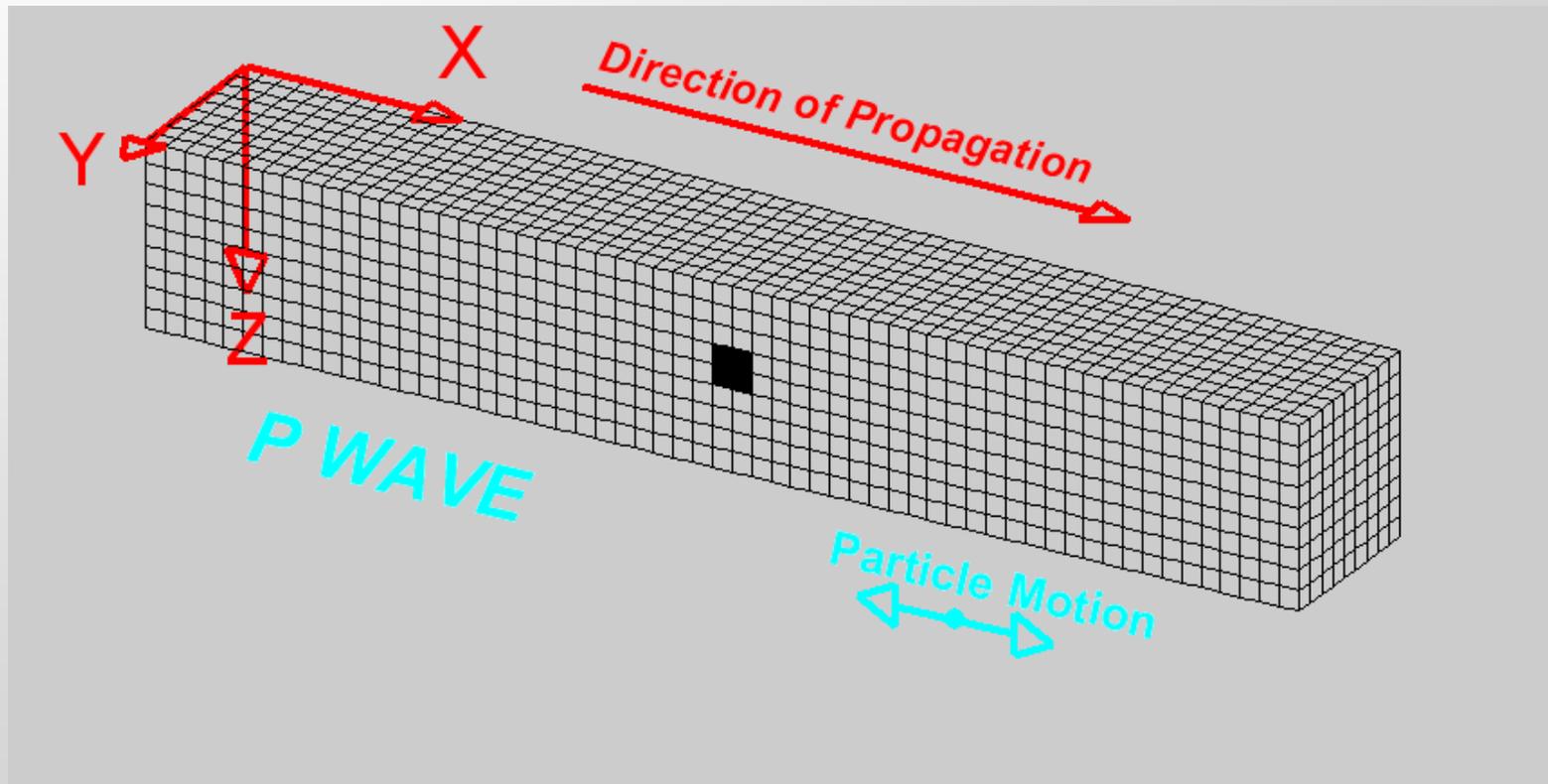
A wave is called **quasi transverse**, if the scalar product  $\mathbf{m} \cdot \mathbf{n} < 0$ .

# 4. Theoretical methods in acoustical studies

## 4.3. Bulk wave propagation in anisotropic elastic media

### 4.3.2. The main equations for bulk waves

#### 4.3.2.6. Visual representation for polarization of P- waves

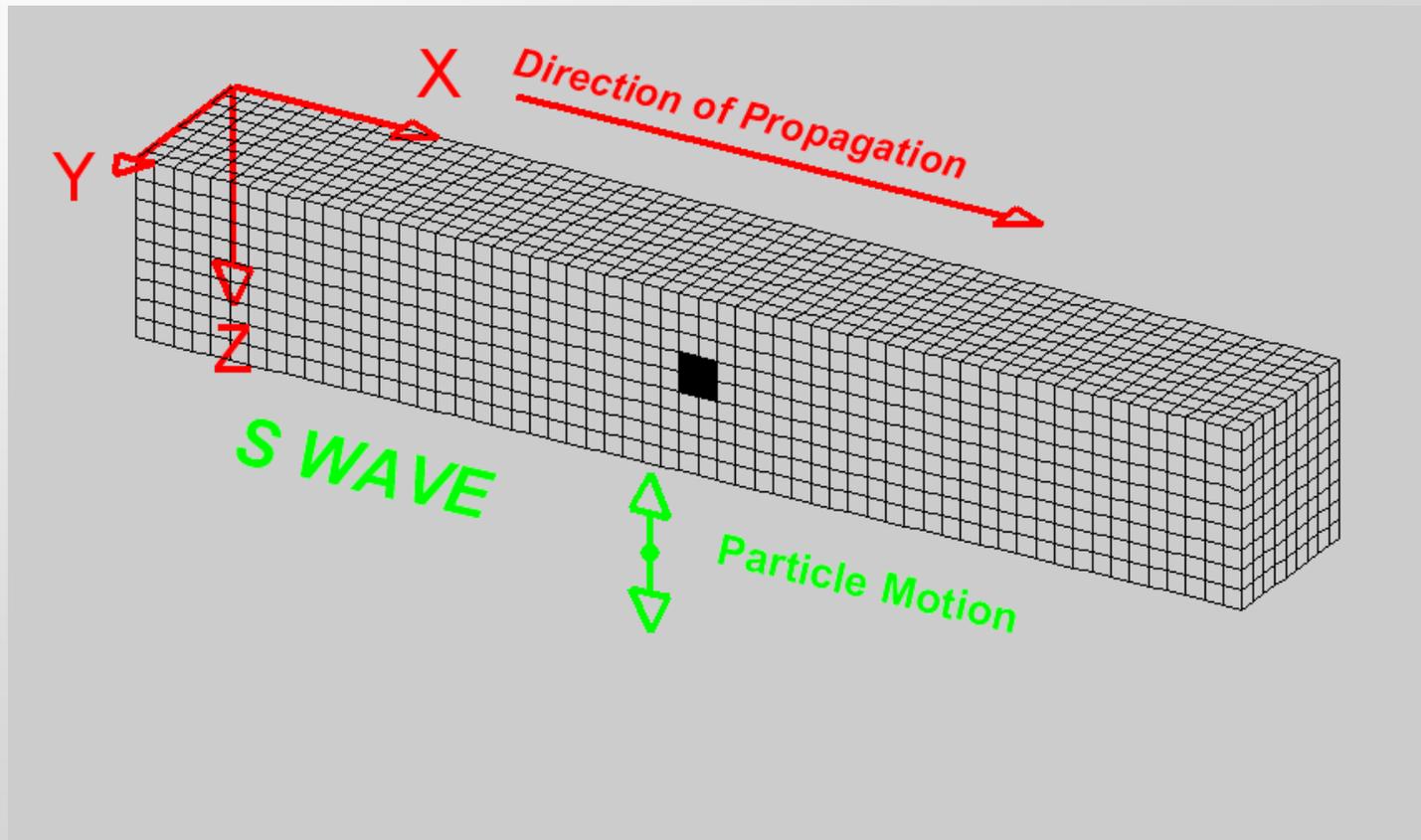


# 4. Theoretical methods in acoustical studies

## 4.3. Bulk wave propagation in anisotropic elastic media

### 4.3.2. The main equations for bulk waves

#### 4.3.2.7. Visual representation for polarization of S- waves



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## 4. Theoretical methods in acoustical studies

### 4.3. Bulk wave propagation in anisotropic elastic media

#### 4.3.2. The main equations for bulk waves

##### 4.3.2.8. Remarks on speeds of bulk waves

#### Remark 1

In most cases speed of the longitudinal (or quasi longitudinal ) wave exceeds speeds of the transverse waves (or quasi transverse), that is why longitudinal waves are called P-waves (Primary waves)

But, there are some exceptions:  $\text{TeO}_2$ , in which one of transverse waves travels faster than the longitudinal

#### Remark 2

In isotropic materials at any admissible Lamé's constants  $\lambda$  and  $\mu$ , speed of transverse waves is (strictly) lower than speed of the longitudinal wave.

## 4. Theoretical methods in acoustical studies

### 4.3. Bulk wave propagation in anisotropic elastic media

#### 4.3.3. The main theorems for bulk waves

##### 4.3.3.1. Theorem on existence of three bulk waves

###### Theorem

For any direction of propagation  $\mathbf{n}$  of an arbitrary anisotropic medium:

- (i) there exist three bulk waves, propagating with (generally) different phase speeds; and
- (ii) having mutually orthogonal polarization vectors;
- (iii) speeds of all bulk waves do not depend upon frequency

###### Remark

While speed of propagation of these bulk waves can coincide, their polarization vectors differ (and they must be mutually orthogonal).

## 4. Theoretical methods in acoustical studies

### 4.3. Bulk wave propagation in anisotropic elastic media

#### 4.3.3. The main theorems for bulk waves

##### 4.3.3.2. Theorem on existence of the acoustic axes

#### Definition

An axis in an anisotropic medium is called “acoustic”, if along it the longitudinal bulk wave can propagate.

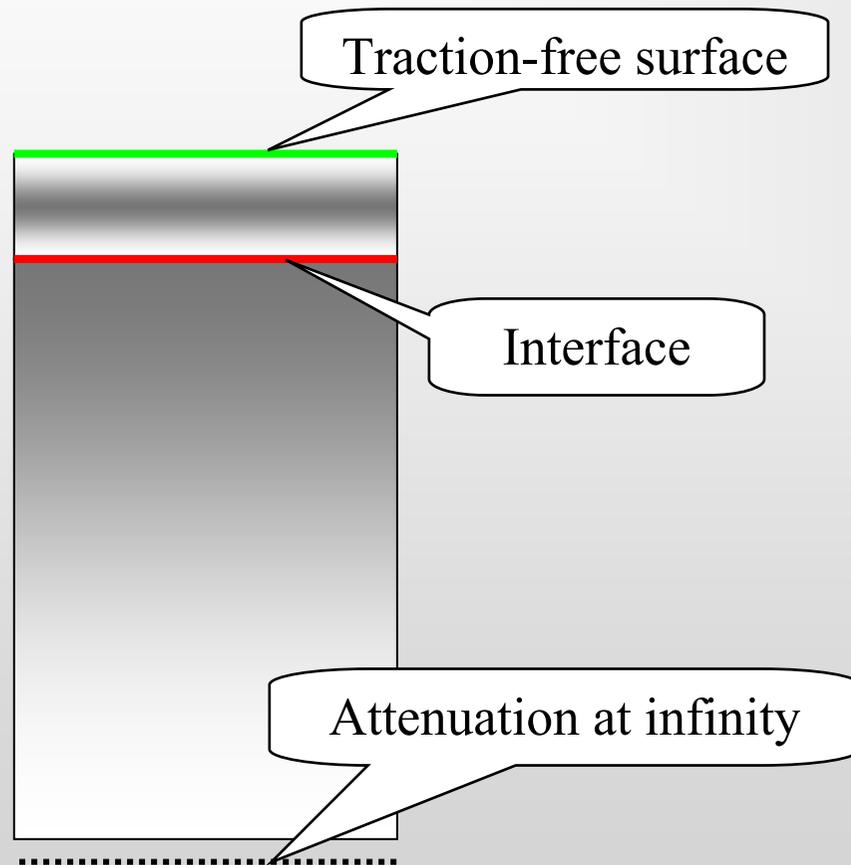
#### Theorem

For any anisotropic medium, there exist at least three different acoustic axes.

# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.1. Boundary, interface, and Sommerfield's conditions



Traction-free surface:

$$\mathbf{t}_{\mathbf{v}} \equiv \mathbf{v} \cdot \mathbf{C} \cdot \nabla \mathbf{u} \Big|_{x'=x_0} = 0$$

Interface:

$$\mathbf{t}_{\mathbf{v}_+}^{upper\ layer} = \mathbf{t}_{\mathbf{v}_-}^{lower\ layer\ (substrate)}$$

$$\mathbf{u}^{upper\ layer} = \mathbf{u}^{lower\ layer\ (substrate)}$$

Sommerfield's attenuation:

$$|\nabla \mathbf{u}(x')| = o(|x'|^{-1}), \quad |x'| \rightarrow \infty$$

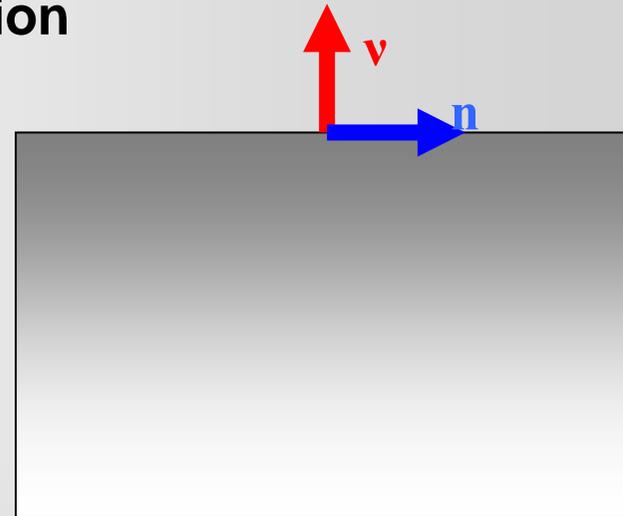
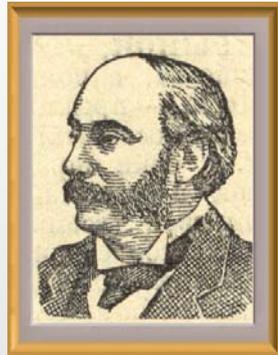
# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.2. Classification of surface acoustic waves

#### 4.4.2.1. Rayleigh waves

##### A. Basic definition



#### Definition

Rayleigh surface wave means attenuating with depth elastic wave with a plane wave front propagating on a traction-free boundary of a half-space (substrate).

#### Remark

The pioneering Rayleigh work, where these waves for the first time were described, appeared in 1885.

## 4. Theoretical methods in acoustical studies

### 4.4. Surface acoustic waves

#### 4.4.2. Classification of surface acoustic waves

##### 4.4.2.1. Rayleigh waves

##### B. The main properties

###### The main theorem

For a long time it was supposed that there can be anisotropic materials (may be artificial), that possesses specific directions along which Rayleigh waves cannot propagate. These hypothetical directions were called “forbidden”.

But, in 1973-1976 Barnett and Lothe proved a theorem **on existence** of Rayleigh wave for any anisotropic materials and any directions.

###### Another property

Rayleigh waves do not possess a dispersion (dependence of frequency on speed).

# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.2. Classification of surface acoustic waves

#### 4.4.2.1. Rayleigh waves

##### C. Role of Rayleigh waves in transmitting energy

These waves play a very important role in transmitting the seismic energy and causing the catastrophic destructions due to the seismic activity.



#### Remark

The amplitude of oscillations of Rayleigh wave attenuates exponentially with depth

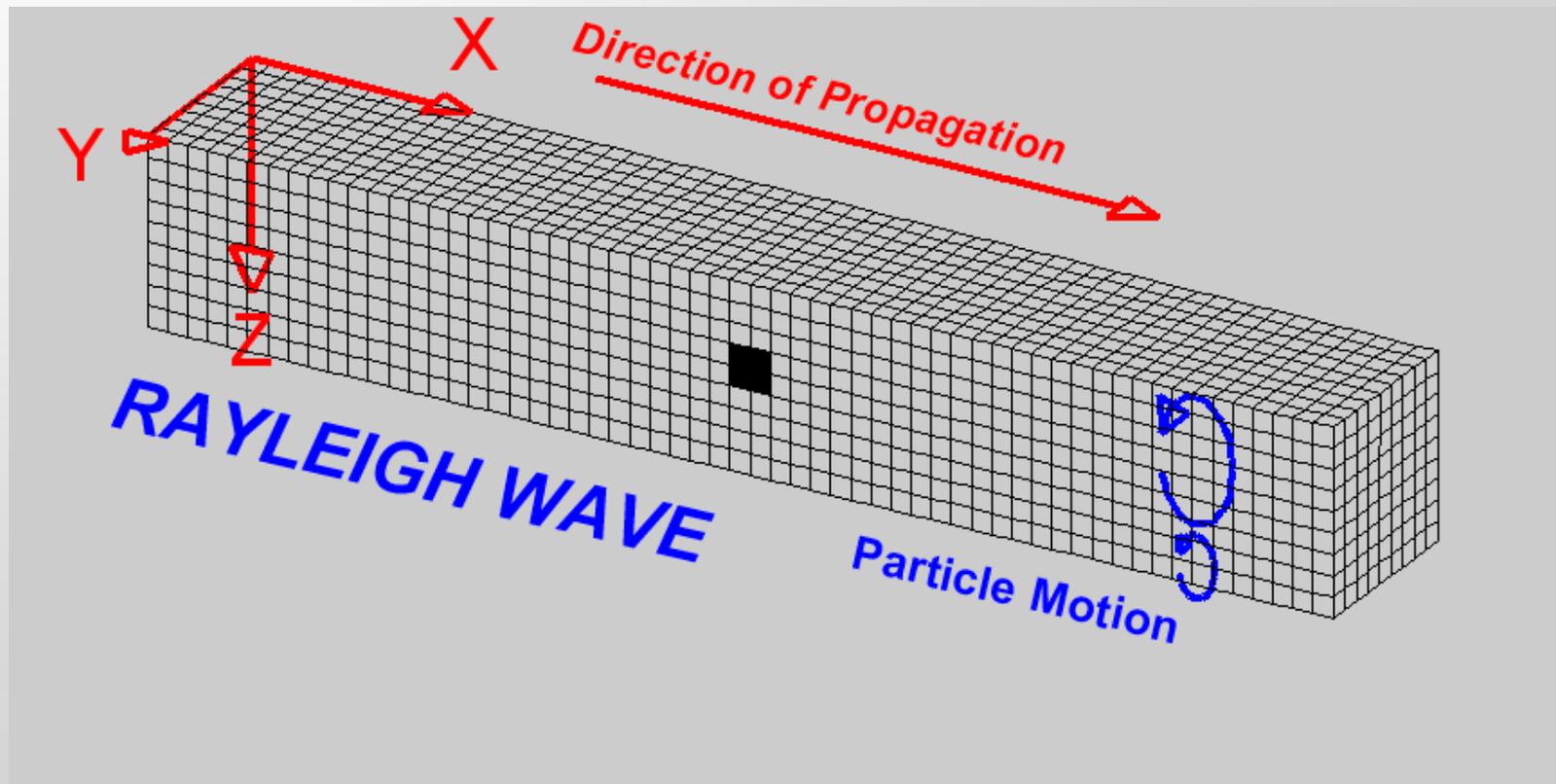
# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.2. Classification of surface acoustic waves

#### 4.4.2.1. Rayleigh waves

#### D. Visualization of Rayleigh wave propagation



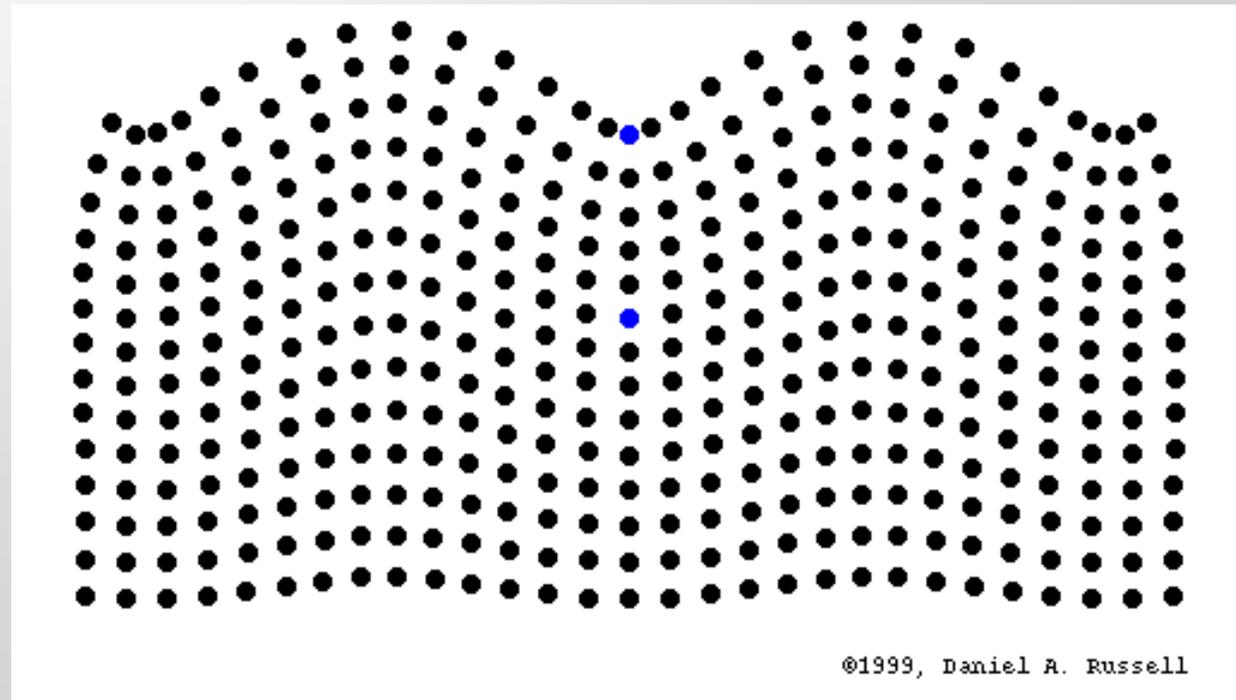
# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.2. Classification of surface acoustic waves

#### 4.4.2.1. Rayleigh waves

#### E. Visualization of particle movements



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# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.2. Classification of surface acoustic waves

#### 4.4.2.1. Rayleigh waves

#### F. Danger for structures



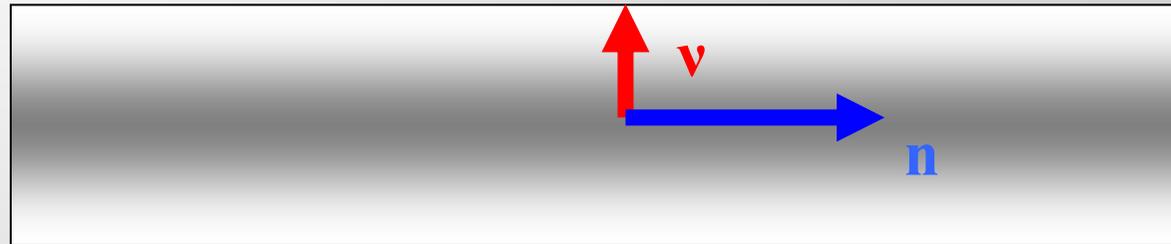
# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.2. Classification of surface acoustic waves

#### 4.4.2.2. Lamb waves

##### A. Basic definition



#### Definition

Lamb waves propagate in a layer with either traction-free, clamped or mixed boundary conditions imposed on the outer surfaces of a layer.

#### Remark

These waves were discovered by Horace Lamb in 1917

## 4. Theoretical methods in acoustical studies

### 4.4. Surface acoustic waves

#### 4.4.2. Classification of surface acoustic waves

##### 4.4.2.2. Lamb waves

###### B. The main properties

- In contrast to Rayleigh waves, Lamb waves are **highly dispersive**, that means the the phase speed depends upon frequency or wavelength.
- There can be an infinite number of Lamb waves propagating with the same phase speed and differing by the frequency.
- Lamb waves can travel with both sub, intermediate, and supersonic speed.

###### Remark

After excitation, the most energy is transferred by the two lowest modes (symmetric and flexural).

# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.2. Classification of surface acoustic waves

#### 4.4.2.2. Lamb waves

#### C. Possible applications

Due to their highly dispersive nature these waves are quite often used in NDT of possible defects in beams, plates, slabs, and rails.

#### Example



Evaluation of defects in rails



Crack determination in rails

# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.2. Classification of surface acoustic waves

#### 4.4.2.3. Stoneley waves

##### A. Basic definition

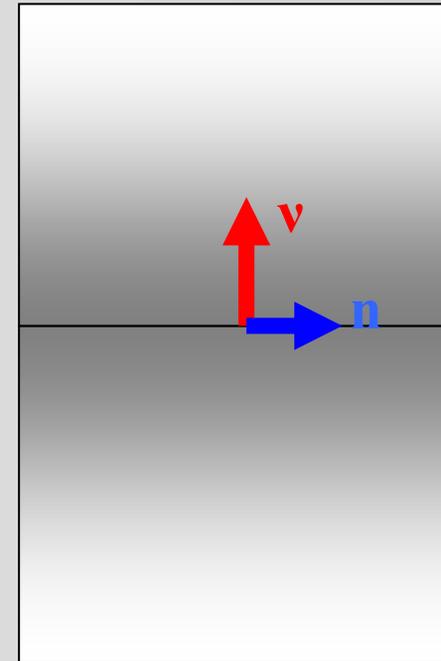


#### Definition

Stoneley waves are the waves traveling on an interface between two contacting half-spaces.

#### Remark

These waves were discovered and described by Robert Stoneley in 1924



## 4. Theoretical methods in acoustical studies

### 4.4. Surface acoustic waves

#### 4.4.2. Classification of surface acoustic waves

##### 4.4.2.3. Stoneley waves

###### B. The main properties

- As Rayleigh waves, Stoneley waves **are not dispersive** (their phase speed does not depend upon frequency or wavelength).
- For Stoneley waves a **uniqueness theorem** can be proved, stating that for arbitrary anisotropic and elastic halfspaces in a contact, there can be no more than one Stoneley wave.
- Generally, Stoneley waves propagate with the subsonic speed.

#### Remark

Not any contacting halfspaces may have Stoneley wave. Conditions for existence for two isotropic halfspaces in a contact were obtained by Stoneley.

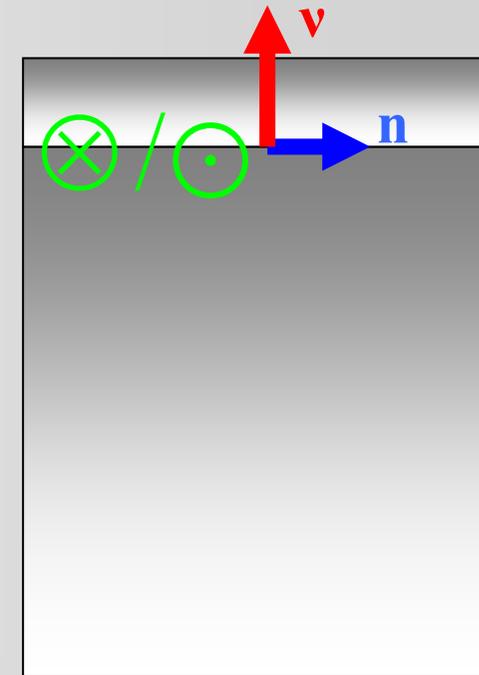
# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.2. Classification of surface acoustic waves

#### 4.4.2.4. Love waves

##### A. Basic definition



#### Definition

Love waves are the waves traveling on an interface between two contacting half-spaces.

#### Remark

These waves were discovered and described by Augustus Love in 1911

Accelerometer on

## 4. Theoretical methods in acoustical studies

### 4.4. Surface acoustic waves

#### 4.4.2. Classification of surface acoustic waves

##### 4.4.2.4. Love waves

##### B. The main properties

- As Lamb waves, Love waves are **highly dispersive**, that means the their phase speed depends upon frequency or wavelength.
- There can be an infinite number of Love waves propagating with the same phase speed and differing by the frequency.
- Love waves can travel with the **subsonic** speeds for the **halfspace**.

#### Remarks

- It is assumed that Love wave attenuates with depth in the halfspace.
- Not any layer and the contacting halfspace may possess Love waves.
- Conditions for existence for both *isotropic* layer and halfspace were obtained by Love.

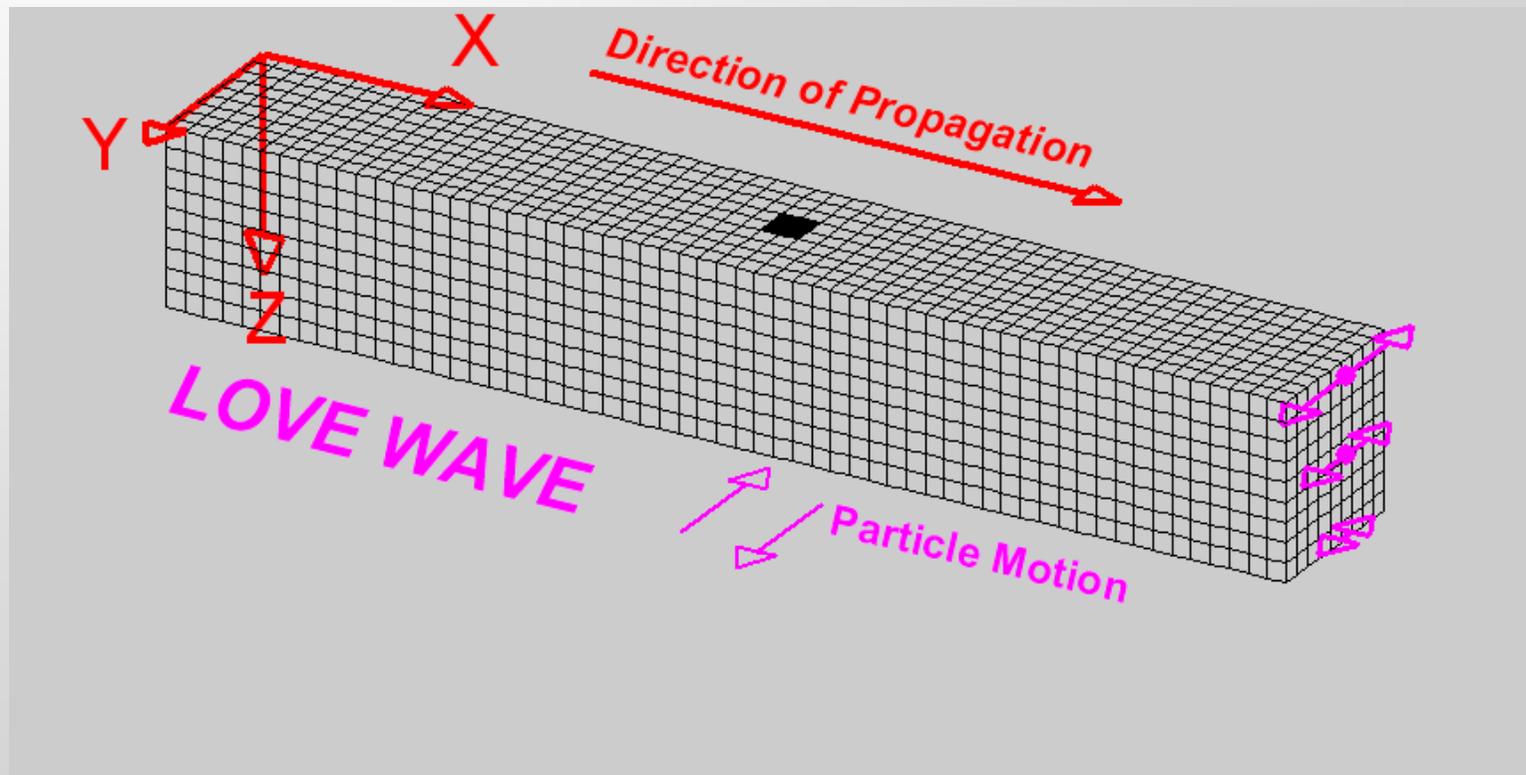
# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.2. Classification of surface acoustic waves

#### 4.4.2.4. Love waves

#### C. Visualization of Love waves



Rail

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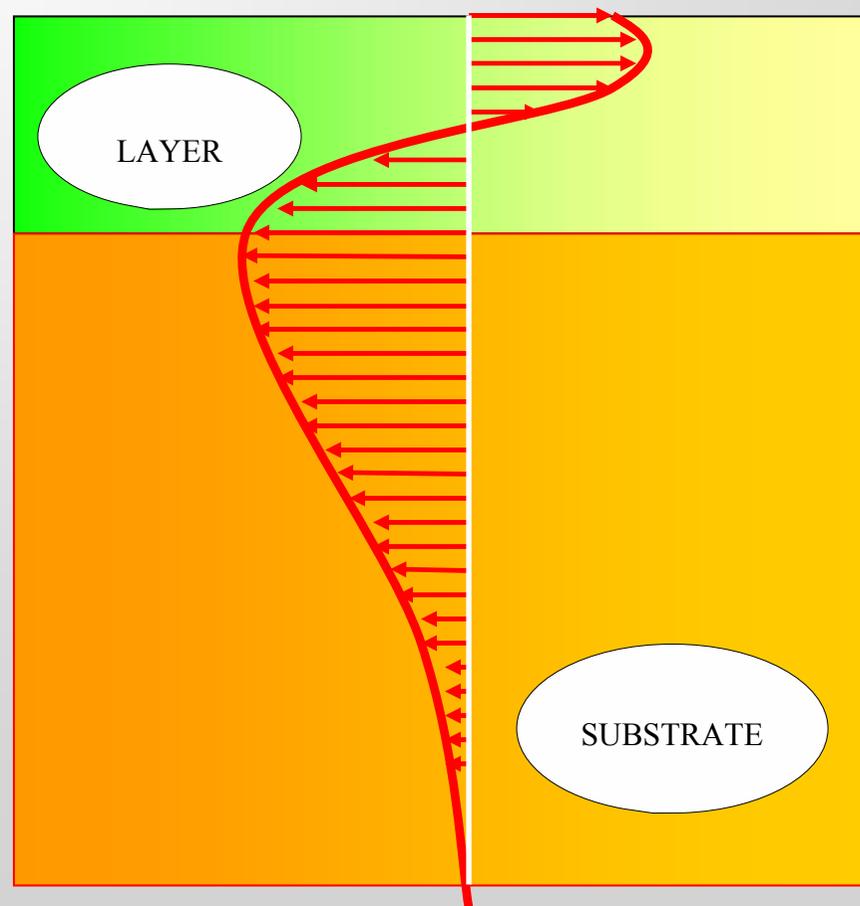
# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.2. Classification of surface acoustic waves

#### 4.4.2.4. Love waves

##### D. Polarization of Love waves



# 4. Theoretical methods in acoustical studies

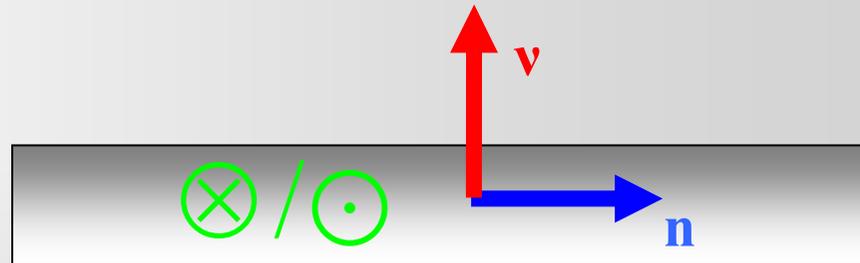
## 4.4. Surface acoustic waves

### 4.4.2. Classification of surface acoustic waves

#### 4.4.2.5. SH waves

##### A. Basic definitions

No  
person is  
associated  
with



#### Definition

These waves travel in a layer or possibly several contacting layers, and have the transverse horizontal polarization.

#### Remark

As Lamb and Love waves, the SH waves are highly dispersive.

Accelerometer on

## 4. Theoretical methods in acoustical studies

### 4.4. Surface acoustic waves

#### 4.4.2. Classification of surface acoustic waves

##### 4.4.2.5. SH waves

##### B. The main properties

- As Lamb and Love waves, SH waves are **highly dispersive**, that means their phase speed depends upon frequency or wavelength.
- There can be an infinite number of SH waves propagating with the same phase speed and differing by the frequency.
- SH waves can travel with both **supersonic** and **subsonic** speed (the subsonic speed cannot be achieved in a layer with the minimal shear bulk wave speed).

Accelerometer on  
Rail

## 4. Theoretical methods in acoustical studies

### 4.4. Surface acoustic waves

#### 4.4.2. Classification of surface acoustic waves

##### 4.4.2.6. Speeds of bulk, Rayleigh, and Love waves

Generally (with some exceptions) the phase speeds satisfy the following conditions:

$$c_{longitudinal}^{bulk} > c_{transverse}^{bulk} > c_{Rayleigh} > c_{Love}^*$$

#### Remark

\* Strictly speaking, there is no single value for Love waves, as these waves are dispersive, and their speed satisfies the condition:

$$\left( c_{transverse}^{bulk} \right)_{layer} < c_{Love} < \left( c_{transverse}^{bulk} \right)_{substrate}$$

It is interesting to note, that there are following relations between the transmitted energy:

$$E_{Love} \sim E_{Rayleigh} \gg E_{bulk}^{Transverse} \sim E_{bulk}^{Longitudinal}$$

# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.3. Mathematical methods for analyzing surface acoustic waves

#### 4.4.3.1. Representations for the displacement field

$$\mathbf{u}(\mathbf{x}, t) = \sum_{k=1}^6 \mathbf{f}_k(x') e^{ir(\mathbf{n} \cdot \mathbf{x} - ct)}$$

Where

$x' = \mathbf{v} \cdot \mathbf{x}$ , thus, it is a coordinate along vector  $\mathbf{v}$

$\mathbf{f}_k$  is the unknown function specifying variation of displacements

$\mathbf{u}$  is a displacement field

$\mathbf{x}$  is a space variable

$t$  is time

$r$  is the wave number ( $r = 2\pi/l$ , or  $r = \omega/c$ )

$\mathbf{n}$  is direction of propagation ( $\mathbf{n}$  is the unit vector)

$c$  is the phase speed

## 4. Theoretical methods in acoustical studies

### 4.4. Surface acoustic waves

#### 4.4.3. Mathematical methods for analyzing surface acoustic waves

##### 4.4.3.2. Christoffel equations

Substituting representation for the surface wave into differential equations of motion, and performing necessary differentiation, yield the Christoffel equations for surface waves:

$$\left[ \mathbf{A}(\mathbf{v}) \partial_{x'}^2 + 2\text{sym}(\mathbf{v} \cdot \mathbf{C} \cdot \mathbf{n}) \partial_{x'} + \mathbf{A}(\mathbf{n}) - \rho c^2 \mathbf{I} \right] \cdot \mathbf{f}(x') = 0$$

Remark

- Thus constructed equation is the **matrix ODE** of the second order
- The unique method of constructing the solution is to reduce this equation to the matrix ODE of the first order.

# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.3. Mathematical methods for analyzing surface acoustic waves

#### 4.4.3.3. Complex six-dimensional formalism

##### A. The main ODE in the complex six-dimensional form

Reducing to the ODE of the first order can be done by introducing a new (vector-valued) function:

$$\mathbf{w}(x') = \partial_{x'} \mathbf{f}(x')$$

Then, the Christoffel equation becomes

$$\partial_{x'} \begin{pmatrix} \mathbf{f} \\ \mathbf{w} \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{A}^{-1}(\mathbf{v}) \cdot (\mathbf{A}(\mathbf{n}) - \rho c^2 \mathbf{I}) & -2\mathbf{A}^{-1}(\mathbf{v}) \cdot \text{sym}(\mathbf{v} \cdot \mathbf{C} \cdot \mathbf{n}) \end{pmatrix} \cdot \begin{pmatrix} \mathbf{f} \\ \mathbf{w} \end{pmatrix}$$

#### Remark

The last first-order ODE gives rise to the six-dimensional complex formalism.

# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.3. Mathematical methods for analyzing surface acoustic waves

#### 4.4.3.3. Complex six-dimensional formalism

##### B. The general solution

This gives 6 linearly independent six dimensional vector-functions, allowing us to construct the general solution:

$$\mathbf{g}_{6\text{-dim}}(x') = \sum_{k=1}^6 C_k \begin{pmatrix} \mathbf{f}_k(x') \\ \mathbf{w}_k(x') \end{pmatrix}$$

#### Remark

The unknown coefficients  $C_k$  are defined by substituting this solution into boundary, interface, and Sommerfield's attenuation conditions

# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.3. Mathematical methods for analyzing surface acoustic waves

#### 4.4.3.3. Complex six-dimensional formalism

##### C. Structure of the solution

$$\mathbf{f}_k(x') = \mathbf{m}_k e^{ir\gamma_k x'}$$

Where

$\mathbf{m}_k$  is the amplitude of the partial wave

$\gamma_k$  is the Christoffel parameter of the partial wave

Remark

The exponential term  $e^{ir\gamma_k x'}$

ensures either exponential growth (if  $\text{Im}(\gamma_k) < 0$ ),

or exponential decay (if  $\text{Im}(\gamma_k) > 0$ ),

or a periodic variation (if  $\text{Im}(\gamma_k) = 0$ ).

# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.3. Mathematical methods for analyzing surface acoustic waves

#### 4.4.3.3. Complex six-dimensional formalism

##### D. History of constructing this formalism

<b>Eshelby</b>	~ 1956	?
<b>Stroh</b>	1962	Development of the sextic formalism
<b>Barnett &amp; Lothe</b>	1973-76	Analysis of Rayleigh waves by the sextic formalism
<b>Chadwick &amp; Smith</b>	1977	Foundations of the sextic formalism
<b>Alshits</b>	1977	Applications to leakage waves
<b>Chadwick &amp; Ting</b>	1987	Structure of the Barnett-Lothe tensors
<b>Mase</b>	1987	Rayleigh wave speed in transversely isotropic media
<b>Ting &amp; Barnett</b>	1997	Classification of surface waves in crystals

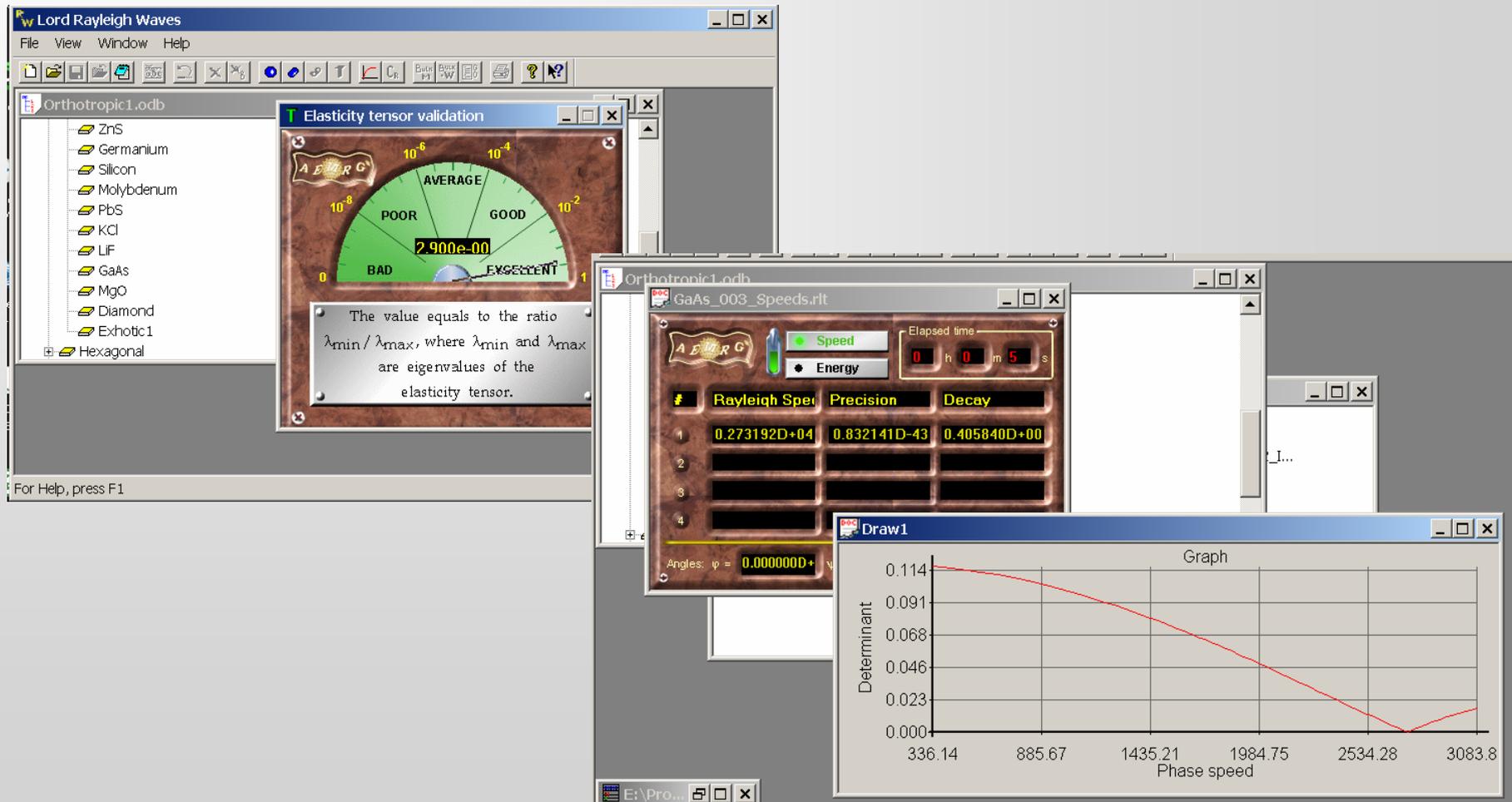
# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.3. Mathematical methods for analyzing surface acoustic waves

#### 4.4.3.4. An example of finding the solutions

##### A. "Lord Rayleigh" Rayleigh wave simulation software



# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.3. Mathematical methods for analyzing surface acoustic waves

#### 4.4.3.4. An example of finding the solutions

##### B. Speed of Rayleigh waves for some materials

Material	Syngony, direction	Rayleigh wave speed, m/sec
GaAs, Gallium Arsenide	Cubic, [001]	2731.8
Ge, Germanium	Cubic, [100]	2929.9
InSb, Indium Antimonide	Cubic, [001]	1833.3

## 4. Theoretical methods in acoustical studies

### 4.4. Surface acoustic waves

#### 4.4.4. Problem of “forbidden” directions for Rayleigh waves

For a long time the main problem related to Rayleigh wave propagating was finding conditions at which such a wave cannot propagate (problem of “forbidden” directions).

Do such “forbidden” directions exist?

Theorem of existence for Rayleigh waves:

Barnett and Lothe, 1973-76

Chadwick, 1975-85

Ting, 1983-96

But, in 1998-2002 a type of Non-Rayleigh waves was theoretically observed and constructed explicitly.

## 4. Theoretical methods in acoustical studies

### 4.4. Surface acoustic waves

#### 4.4.5. Anomalous solutions for Rayleigh waves (Non-Rayleigh wave type waves)

##### 4.4.5.1. Conditions for appearing the anomalous waves

These waves correspond to appearing the **Jordan blocks** in a six-dimensional matrix associated with the Christoffel equation.

Structure of a Non-Rayleigh type wave

$$\mathbf{u}(\mathbf{x}, t) = \left( \mathbf{m}_1 + \mathbf{m}_2^* x' \right) e^{ir\gamma x'} e^{ir(\mathbf{n} \cdot \mathbf{x} - ct)}$$

Where

$\mathbf{m}_2^*$  is the generalized eigenvector

# 4. Theoretical methods in acoustical studies

## 4.4. Surface acoustic waves

### 4.4.5. Anomalous solutions for Rayleigh waves (Non-Rayleigh wave type waves)

#### 4.4.5.2. Jordan blocks

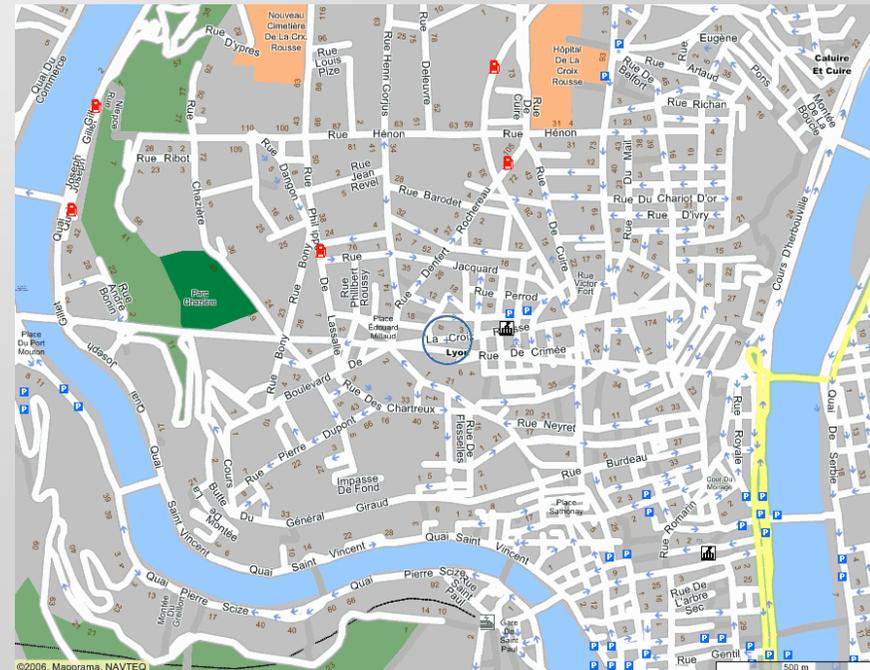


$$\mathbf{J} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Marie Ennemond Camille

Jordan

1838 - 1922





## 4. Theoretical methods in acoustical studies

### 4.5. Surface acoustic waves in multilayered media

#### 4.5.2. Transfer matrix method

##### The main idea

Constructing the “transfer” matrix allowing us to express boundary conditions at the bottom boundary in terms of the coefficients of the uppermost layer

$$M \equiv A_1^+ \cdot (A_1^-)^{-1} \cdot A_2^+ \cdot (A_2^-)^{-1} \cdot \dots \cdot A_n^+ \quad \Rightarrow \quad \det(M) = 0$$

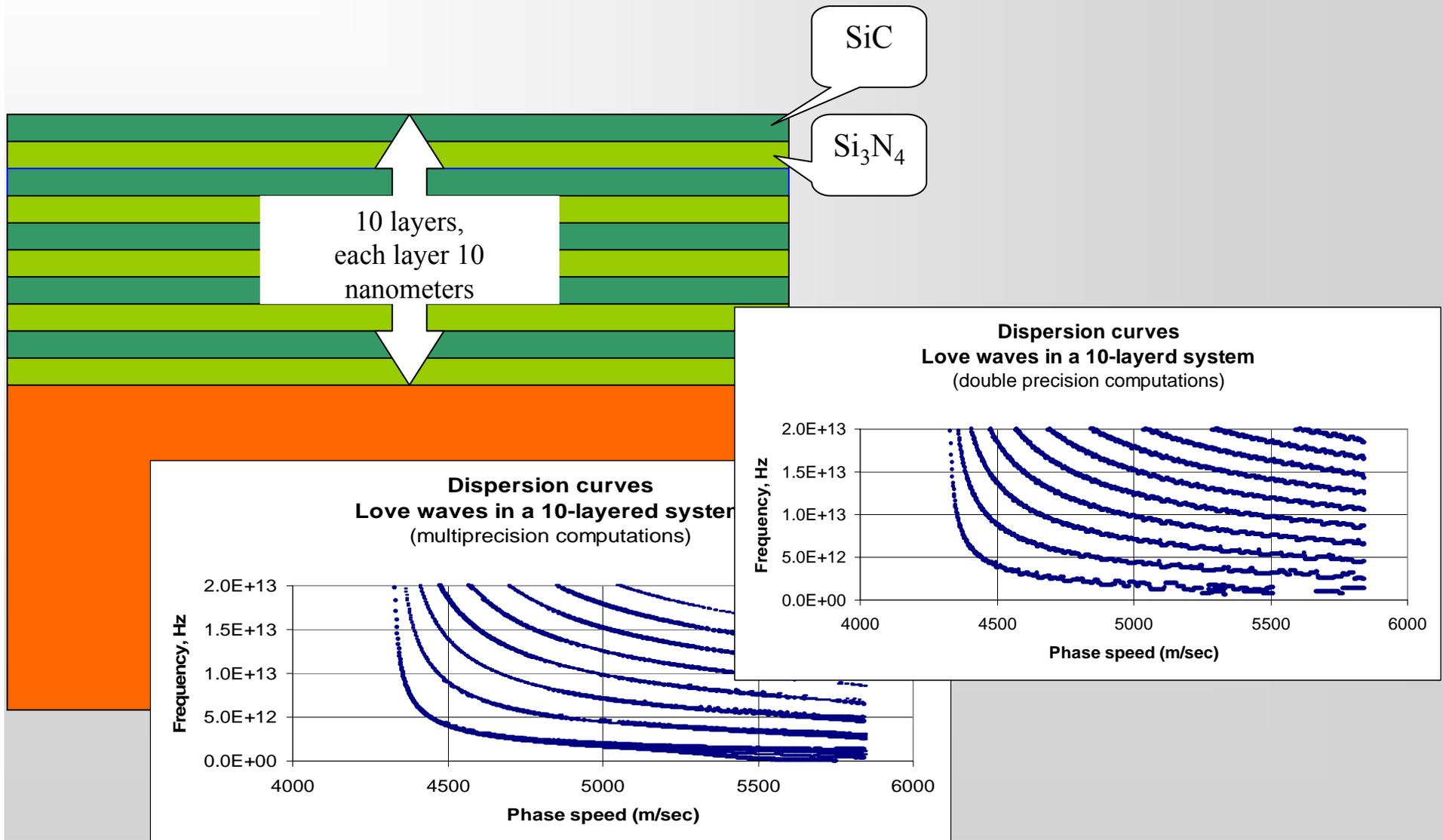
##### Originator

Suggested by Thomson (1950) and Haskell (1953)

# 4. Theoretical methods in acoustical studies

## 4.5. Surface acoustic waves in multilayered media

### 4.5.3. Role of multiprecision computations

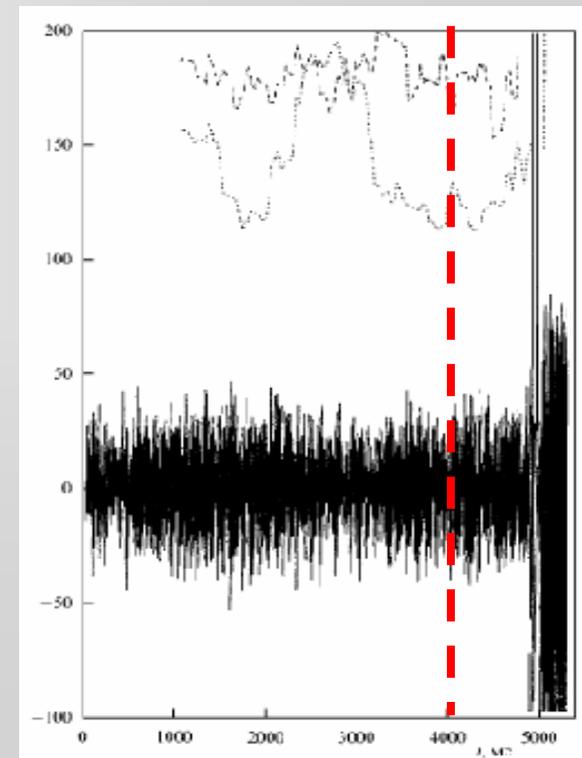
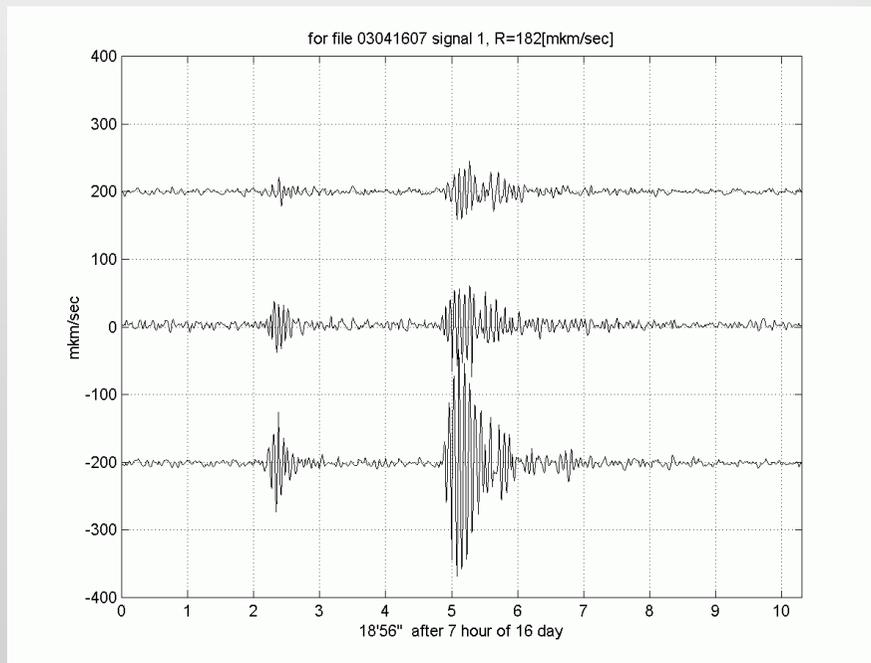


## Part III. Engineering applications

# 5. Deterministic mathematical methods used for predicting earthquakes

## 5.1. The main problem of predicting analysis

Analysis of a seismogram  $\Rightarrow$  conclusion on possibility of the event



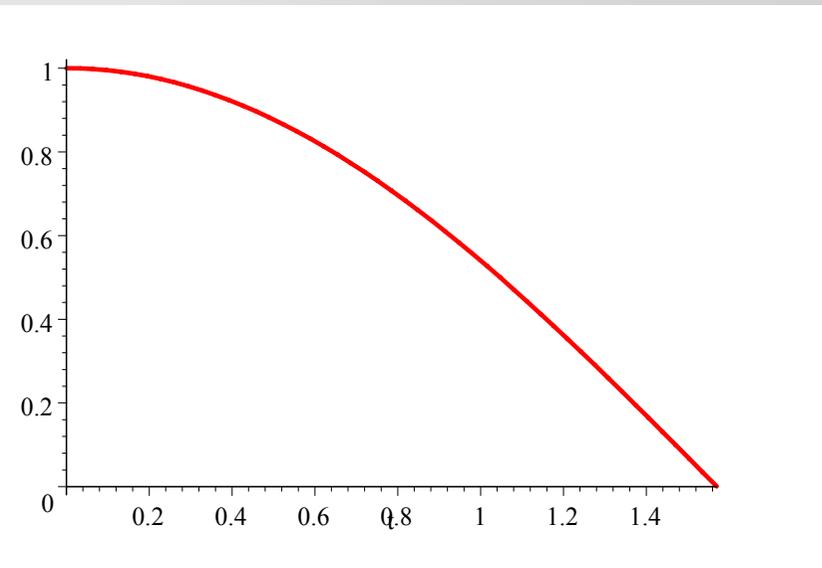
# 5. Deterministic mathematical methods used for predicting earthquakes

## 5.2. Fourier and wavelet analyses

### 5.2.1. Fourier transforms

#### 5.2.1.1. Example of a function to be extrapolated

$$\cos(t) \text{ at } t \in [0, \pi/2]$$



# 5. Deterministic mathematical methods used for predicting earthquakes

## 5.2. Fourier and wavelet analyses

### 5.2.1. Fourier transforms

#### 5.2.1.2. Fourier analysis and synthesis

$$f(t) \sim \sum_{k=-\infty}^{\infty} c_k \exp(2\pi i k t / p)$$

**Where**

$$c_k = \frac{1}{p} \int_{t_0}^{t_0+p} f(\tau) \exp(2\pi i k \tau / p) d\tau$$

**Fourier series for the given function**

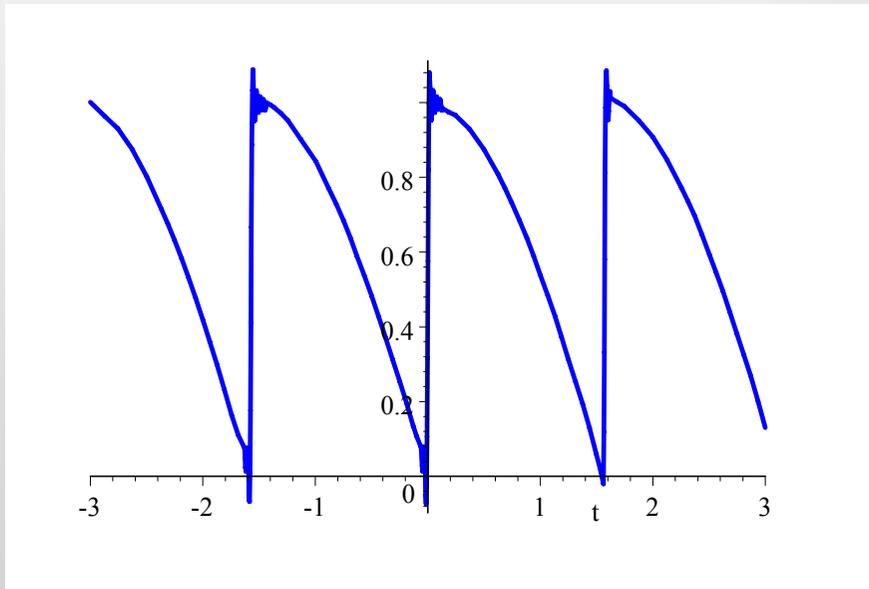
$$f(t) \sim \sum_{k=-\infty}^{\infty} \frac{2(4ik - \exp(2ik\pi))}{\pi(16k^2 - 1)} \exp(4i kt)$$

# 5. Deterministic mathematical methods used for predicting earthquakes

## 5.2. Fourier and wavelet analyses

### 5.2.1. Fourier transforms

#### 5.2.1.3. Resulting function



Summation of Fourier series  
for following initial function

$$\cos(t) \text{ at } t \in [0, \pi/2]$$

# 5. Deterministic mathematical methods used for predicting earthquakes

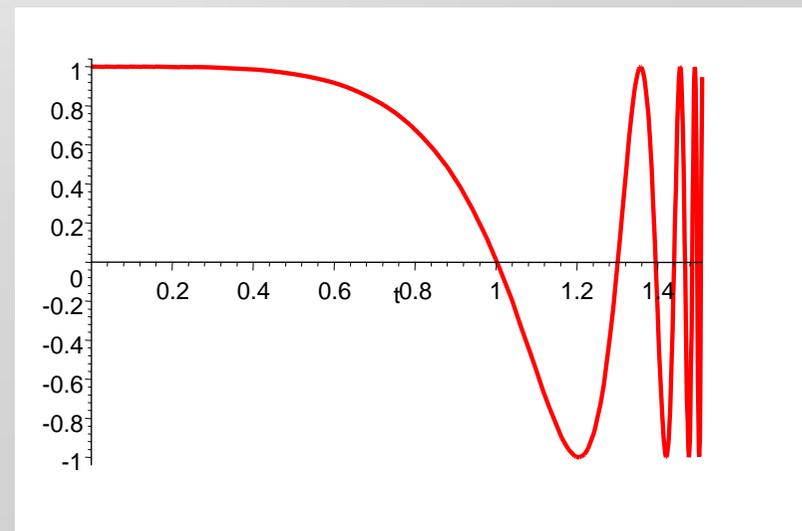
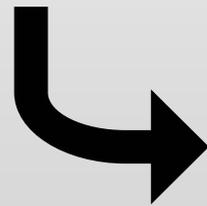
## 5.2. Fourier and wavelet analyses

### 5.2.1. Fourier transforms

#### 5.2.1.4. Shortcomings

Fourier analysis (series) is inapplicable to predicting:

- non-periodic processes
- processes with the unknown period(s)
- processes with variable periods



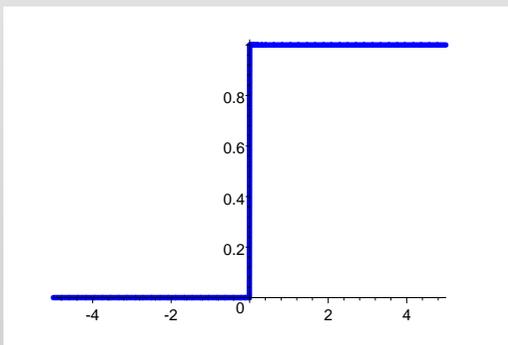
# 5. Deterministic mathematical methods used for predicting earthquakes

## 5.2. Fourier and wavelet analyses

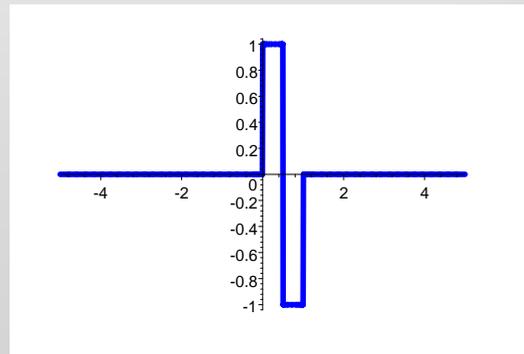
### 5.2.2. Wavelet transforms

#### 5.2.2.1. The main ideas

Heaviside step function



Haar's generic wavelet



Wavelets

$$\psi_{j,k}(t) = 2^{j/2} \psi_H(2^j t - k)$$

$$\psi_H(t) = H(t)H(1-2t) - H(2t-1)H(1-t)$$

# 5. Deterministic mathematical methods used for predicting earthquakes

## 5.2. Fourier and wavelet analyses

### 5.2.2. Wavelet transforms

#### 5.2.2.2. Basic properties

**Orthogonality:**

$$\int \psi_{j,k}(t) \psi_{m,n}(t) dt = 0, \quad \text{at } j \neq m \quad \underline{\underline{\text{or}}} \quad k \neq n$$

**Normality:**

$$\int (\psi_{j,k}(t))^2 dt = 1$$

# **5. Deterministic mathematical methods used for predicting earthquakes**

## **5.2. Fourier and wavelet analyses**

### **5.2.2. Wavelet transforms**

#### **5.2.2.3. Shortcomings and advantage**

#### **Shortcomings:**

Wavelet analysis is inapplicable to predicting:

- non-periodic processes
- processes with the unknown period(s)

#### **Advantage:**

Wavelet analysis is well suited for processes with the variable periods

# 5. Deterministic mathematical methods used for predicting earthquakes

## 5.3. Lagrange and Newton interpolating polynomials

### 5.3.1. Basic idea (assumption of analyticity)

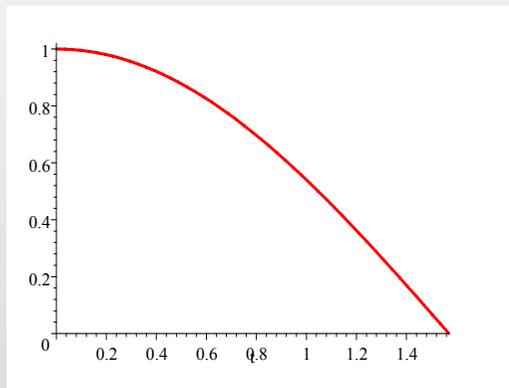
Assume that the function to be extrapolated is analytic in a vicinity of the endpoint, then such a function can be expanded into convergent Taylor's series:

$$f(t) = \sum_{k=0}^{\infty} \frac{f^{(k)}(t_b)}{k!} (t - t_b)^k$$

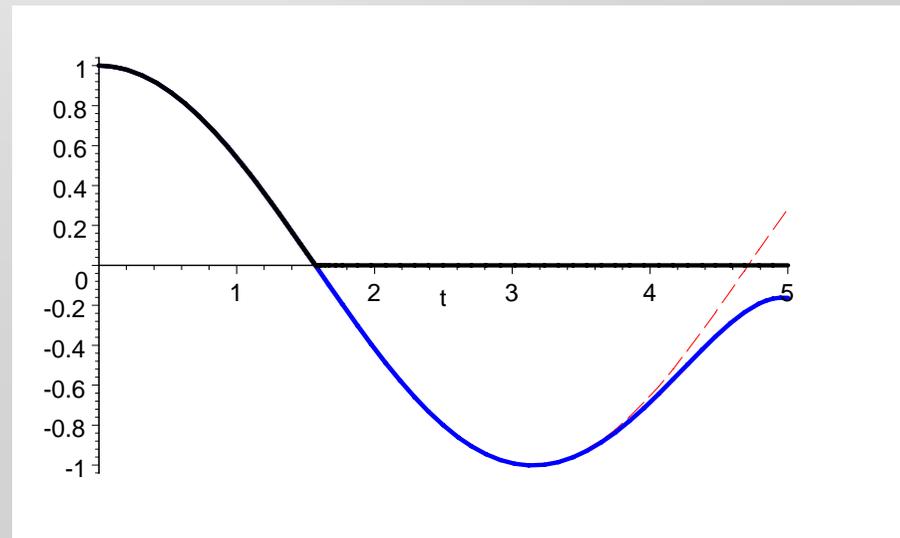
# 5. Deterministic mathematical methods used for predicting earthquakes

## 5.3. Lagrange and Newton interpolating polynomials

### 5.3.2. An example of cosine function



Our cosine function at the endpoint  $\pi/2$  can be expanded into Taylor's series, which after truncating to the first 10 terms gives the following:



# 5. Deterministic mathematical methods used for predicting earthquakes

## 5.3. Lagrange and Newton interpolating polynomials

### 5.3.3. Transition to interpolating polynomials

Unfortunately, in most of practical situations we do not know analytical expressions for the function to be interpolated

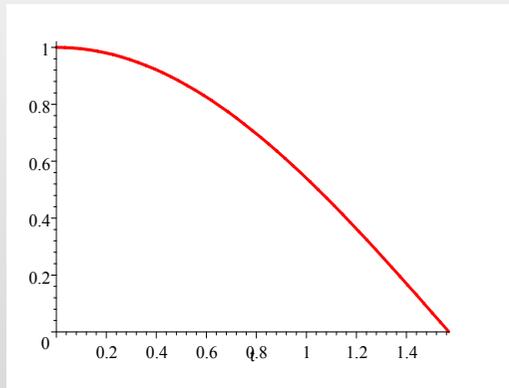


But, we can try to construct an interpolating polynomial, and then find extrapolation by the interpolating polynomial

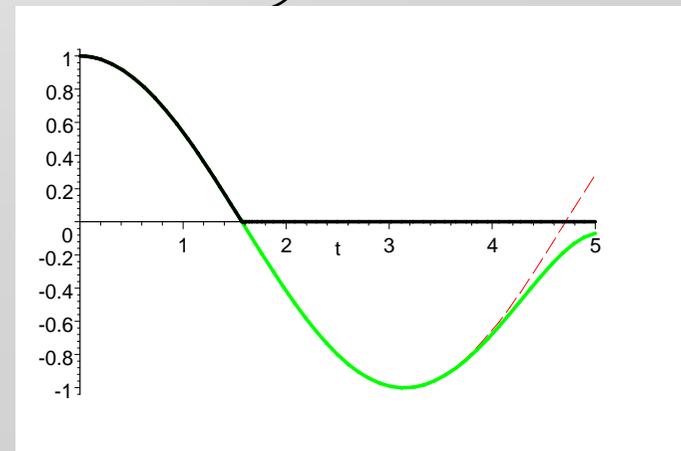
# 5. Deterministic mathematical methods used for predicting earthquakes

## 5.3. Lagrange and Newton interpolating polynomials

### 5.3.4. Interpolating polynomial for cosine function



Lagrange interpolating polynomial of the 10-th order



# 5. Deterministic mathematical methods used for predicting earthquakes

## 5.4. Role of multiprecision calculations in the predicting analyses

### 5.4.1. An example of a two-term polynomial

$$P(x) = x^{10} - 10x^9$$

Roots:

$$x^9(x - 10) = 0 \quad \Rightarrow \quad x_1 = 0, \quad x_2 = 10$$

# 5. Deterministic mathematical methods used for predicting earthquakes

## 5.4. Role of multiprecision calculations in the predicting analyses

### 5.4.2. A small perturbation

$$P_{\varepsilon}(x) = x^{10} - 10.00001x^9$$



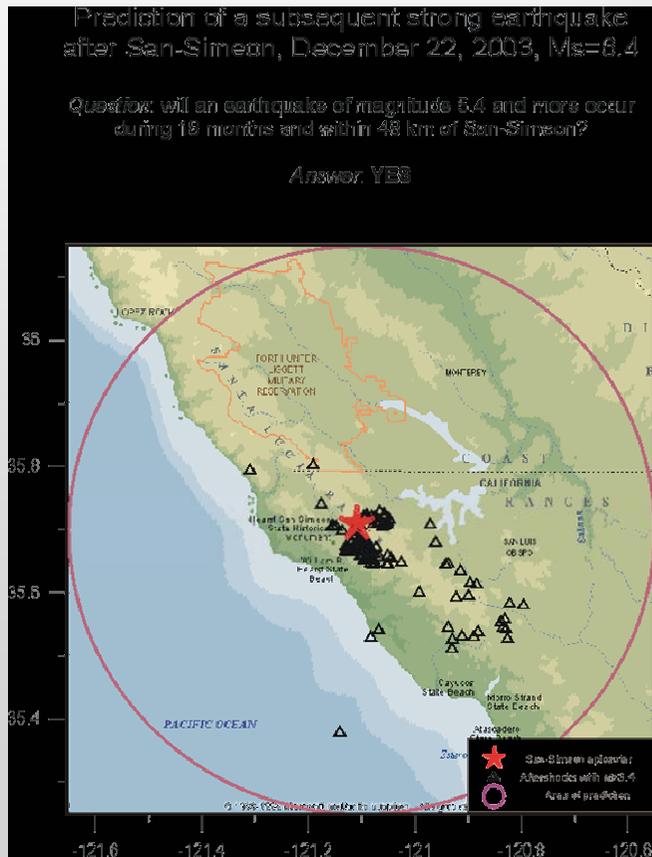
$$P_{\varepsilon}(10) = -10000$$

while

$$P(10) = 0$$

# 5. Deterministic mathematical methods used for predicting earthquakes

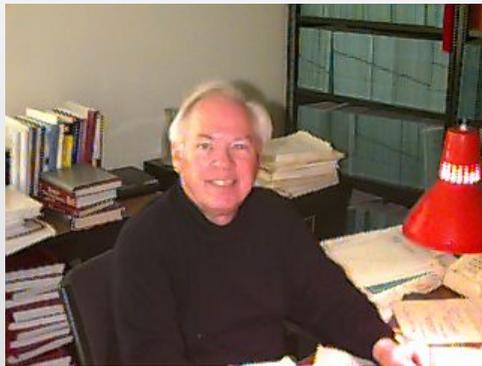
## 5.5. Example of predicting earthquakes in US



Prediction dated Dec 23 2003 is based on the analysis of small vibrations in the San-Simeon region (CA)

# 6. Principles of creating seismic and vibration barriers

## 6.1. Rough surface, as a barrier for Rayleigh waves



Alexei A. Maradudin

Works on rough surfaces for Rayleigh waves 1976-78

### The main results:

If a free surface is not flat, but contains some small periodic perturbations (of Lyapunov class), then the corresponding Rayleigh wave begins attenuate

The rate of attenuation depends upon frequency of Rayleigh wave

## 6. Principles of creating seismic and vibration barriers

### 6.2. Modifying surface layers for creating barriers against Love waves

#### 6.2.1. The main principle (Actually, A.E.H.Love, 1911):

Love wave cannot propagate in an isotropic elastic layer perfectly connected to the isotropic elastic halfspace, if speed of propagation of bulk shear wave in the layer is greater than in the substrate:

$$\left( c_{transverse}^{bulk} \right)^{layer} > \left( c_{transverse}^{bulk} \right)^{substrate}$$

## 6. Principles of creating seismic and vibration barriers

### 6.2. Modifying surface layers for creating barriers against Love waves

#### 6.2.2. Consequence

$$\left. \begin{array}{l} c_{transverse}^{bulk} = \sqrt{\frac{\mu}{\rho}} \\ \& \\ \text{Love's principle} \end{array} \right\}$$



$$\frac{\mu^{layer}}{\mu^{substrate}} > \frac{\rho^{layer}}{\rho^{substrate}}$$



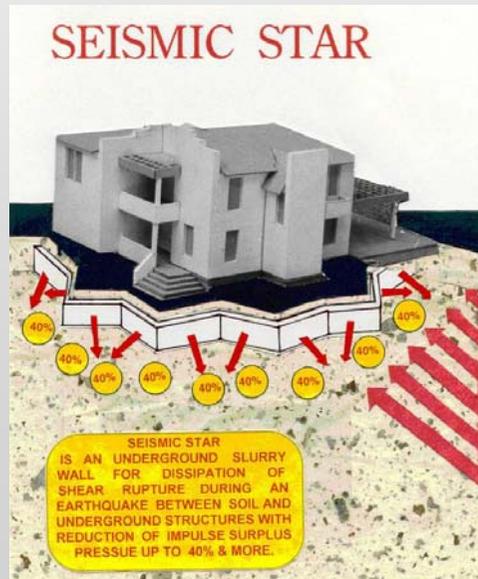
A condition for a Love wave barrier

## 6. Principles of creating seismic and vibration barriers

### 6.3. “Walls” in rocks and soils to prevent surface acoustic waves to propagate

#### 6.3.1. Principle of reflection

##### 6.3.1.1. An example of the reflecting barrier



Kalmatron Corp. with its star-shaped protection system

One of the obvious deficiencies:

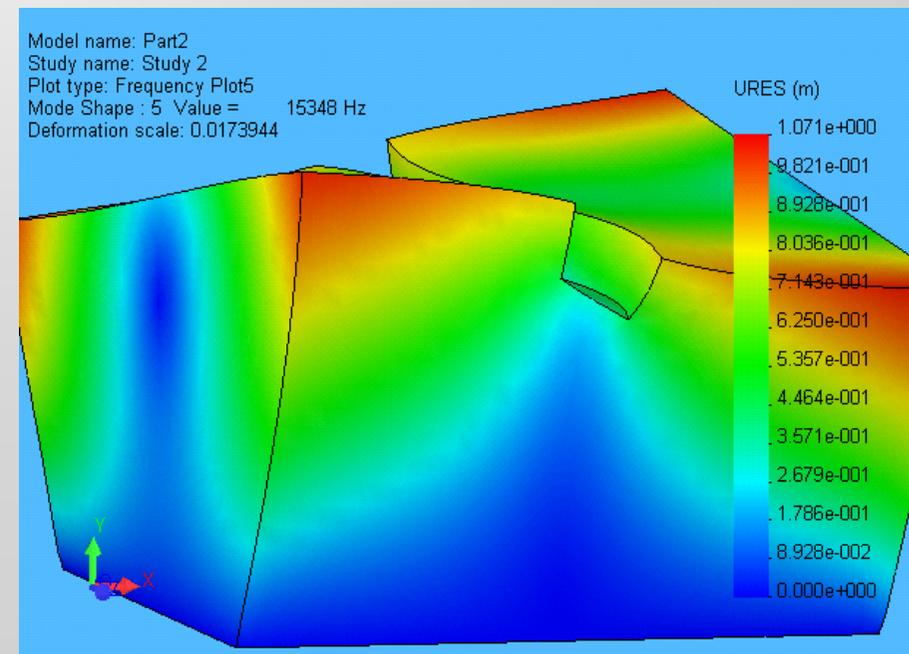
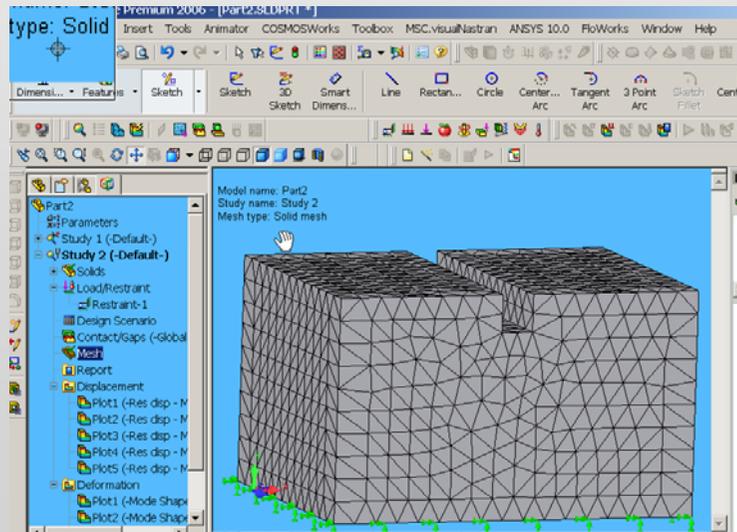
For a relatively large wavelength of seismic waves (10-6000m) the protected system should be at least the same depth

# 6. Principles of creating seismic and vibration barriers

## 6.3. “Walls” in rocks and soils to prevent surface acoustic waves to propagate

### 6.3.1. Principle of reflection

#### 6.3.1.2. Some problems in creating reflecting seismic barriers

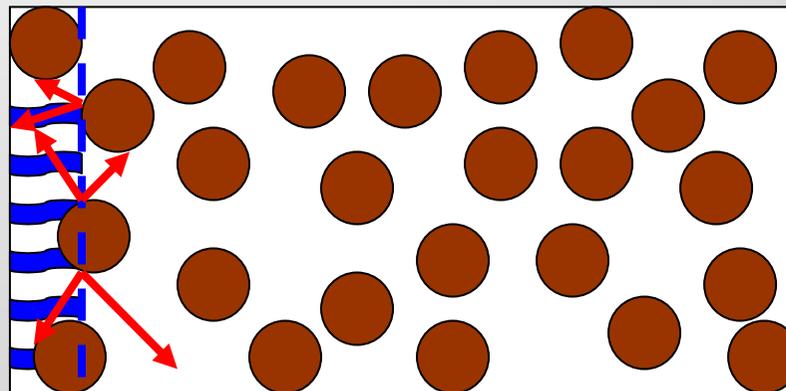


## 6. Principles of creating seismic and vibration barriers

6.3. “Walls” in rocks and soils to prevent surface acoustic waves to propagate

6.3.2. Principle of scattering

6.3.2.1. The main idea



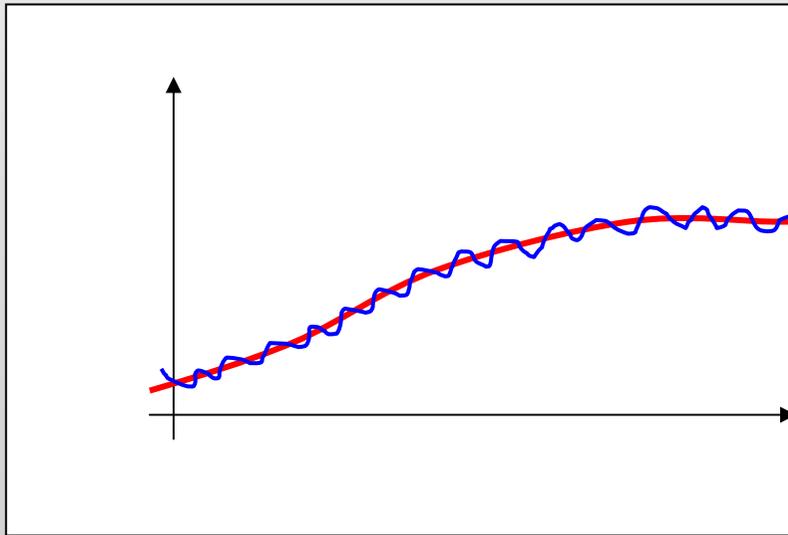
The best results with respect to scattering can be achieved, if inclusions are the closed pores.

# 6. Principles of creating seismic and vibration barriers

6.3. "Walls" in rocks and soils to prevent surface acoustic waves to propagate

6.3.2. Principle of scattering

6.3.2.2. Mathematical method



Two-scale asymptotic analysis

$x$  – "slow" variable

$X$  – "fast" variable

Relation between variables

$$X = \frac{1}{\varepsilon} x, \quad \varepsilon \rightarrow 0$$

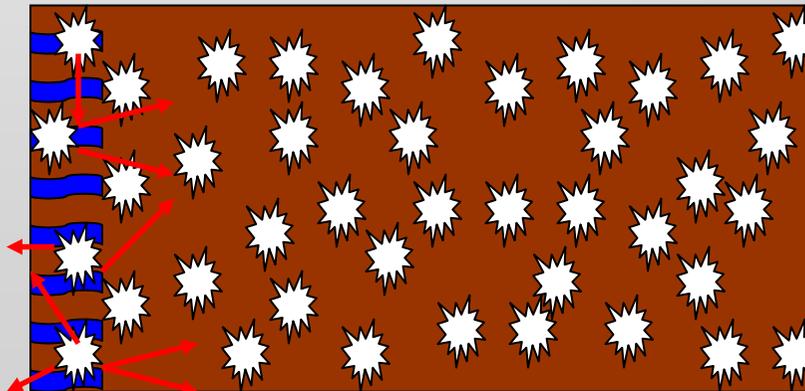
# 6. Principles of creating seismic and vibration barriers

6.3. “Walls” in rocks and soils to prevent surface acoustic waves to propagate

6.3.2. Principle of scattering

6.3.2.3. The main result

The best results with respect to the scattering effect are achieved, when the inclusions are pores



# 7. Viscoelastic dampers for vibration and seismic protection

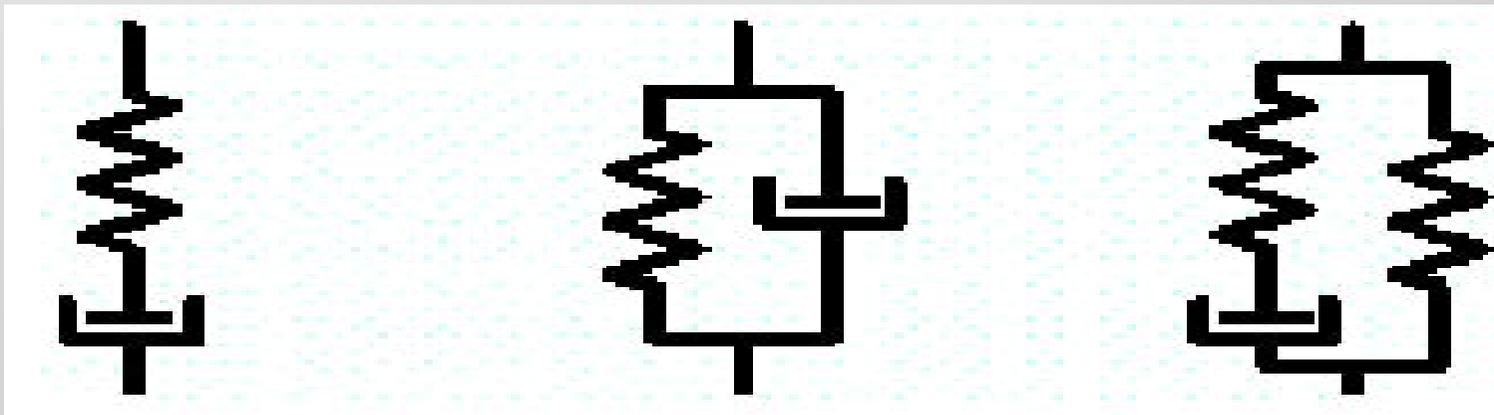
## 7.1. Maxwell, Kelvin (Voigt), and standard elements

### 7.1.1. The main elements

Maxwell element

Kelvin (Voigt) element

Standard linear



# 7. Viscoelastic dampers for vibration and seismic protection

## 7.1. Maxwell and Kelvin (Voigt) elements

### 7.1.2. Differential equation for Kelvin's element

$$m\ddot{x} + c\dot{x} + kx = 0$$

$m$  is mass

$c$  is viscosity of a dashpot

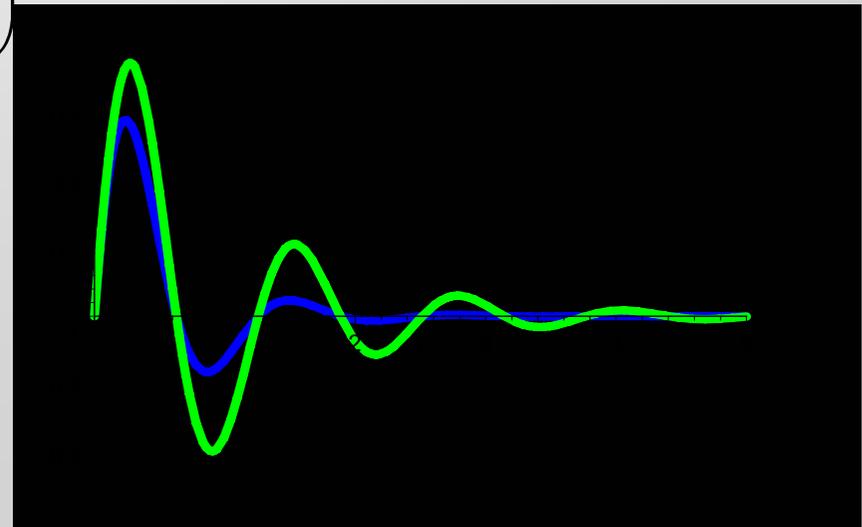
$k$  is the spring rate

# 7. Viscoelastic dampers for vibration and seismic protection

## 7.1. Maxwell and Kelvin (Voigt) elements

### 7.1.3. The general solution of the equation for Kelvin's element (free vibrations)

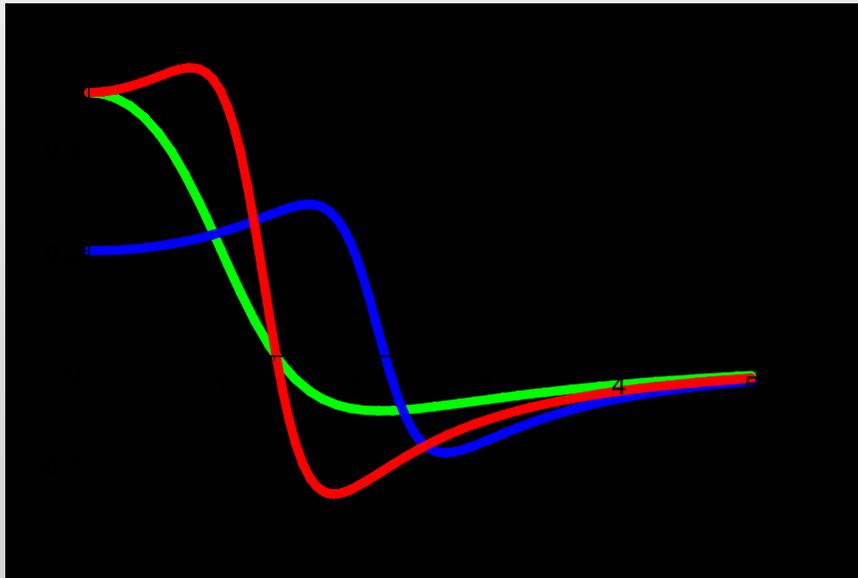
$$x = \exp\left(-\frac{c}{2m}t\right) \exp\left(i\sqrt{\frac{k}{m} - \left(\frac{c}{2m}\right)^2}t\right)$$



# 7. Viscoelastic dampers for vibration and seismic protection

## 7.1. Maxwell and Kelvin (Voigt) elements

### 7.1.4. Response to the oscillating loadings

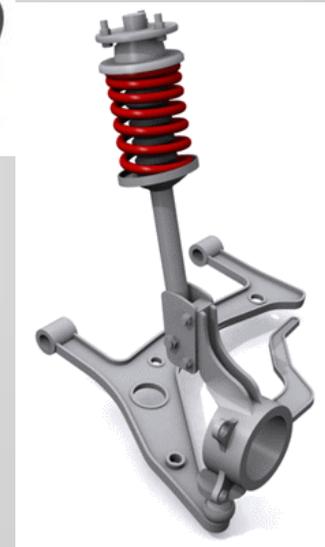
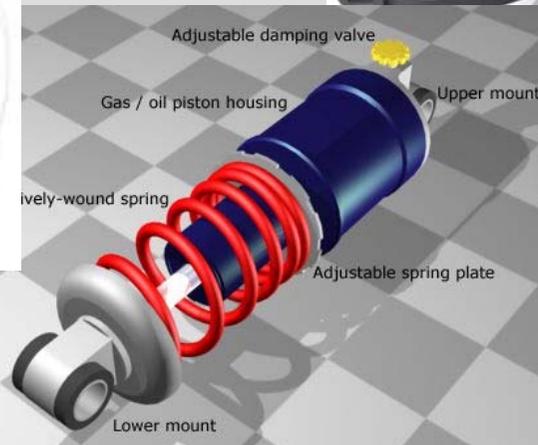
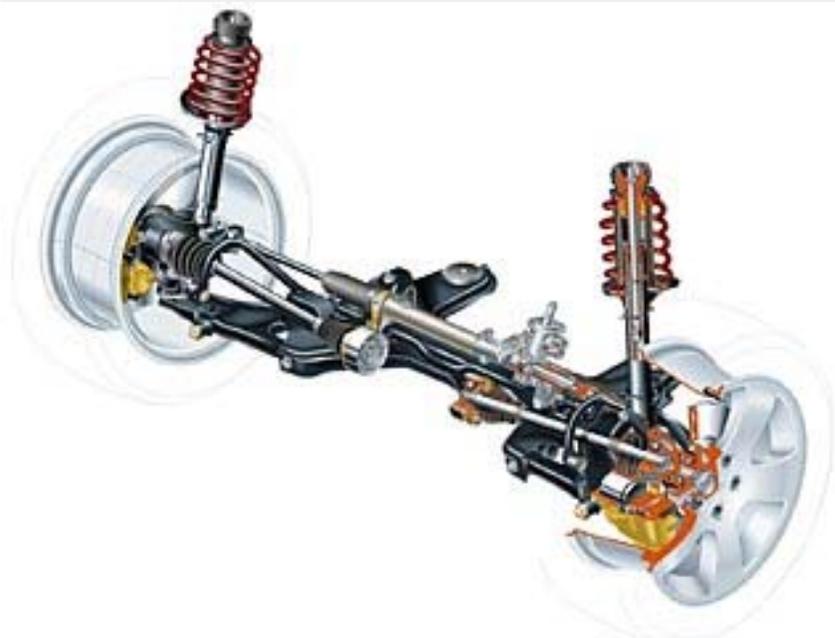


Dependence of the amplitude of oscillations upon frequency of the applied loading for the fixed damping system (Kelvin's element)

# 7. Viscoelastic dampers for vibration and seismic protection

## 7.2. Damping in automotive industry

### 7.2.1. Design of McPherson struts and suspensions

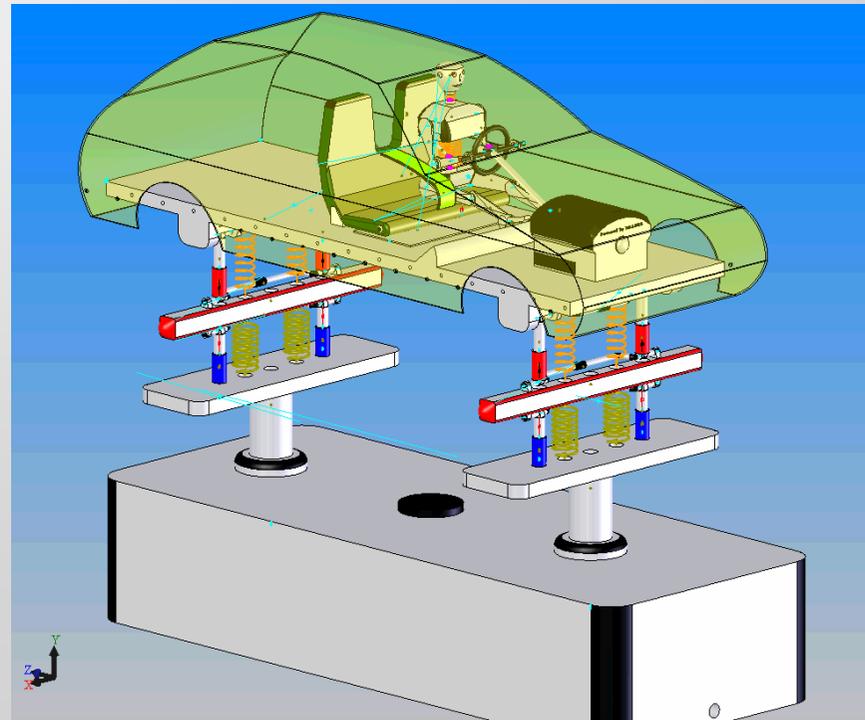


Derived from the "Bible of Car Suspensions"

# 7. Viscoelastic dampers for vibration and seismic protection

## 7.2. Damping in automotive industry

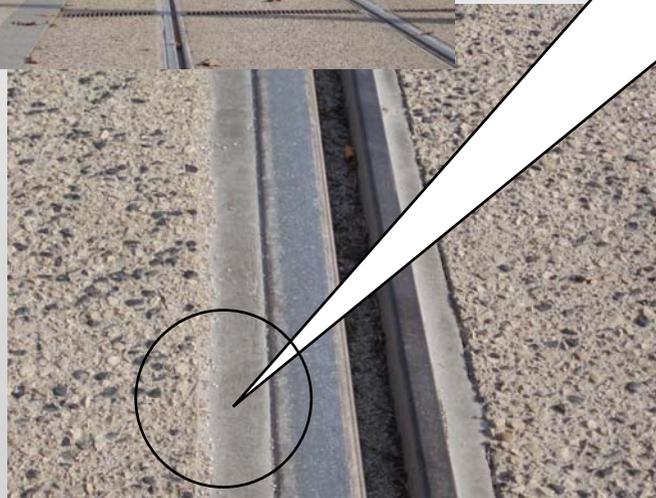
### 7.2.2. Example of unsuccessful suspension tuning



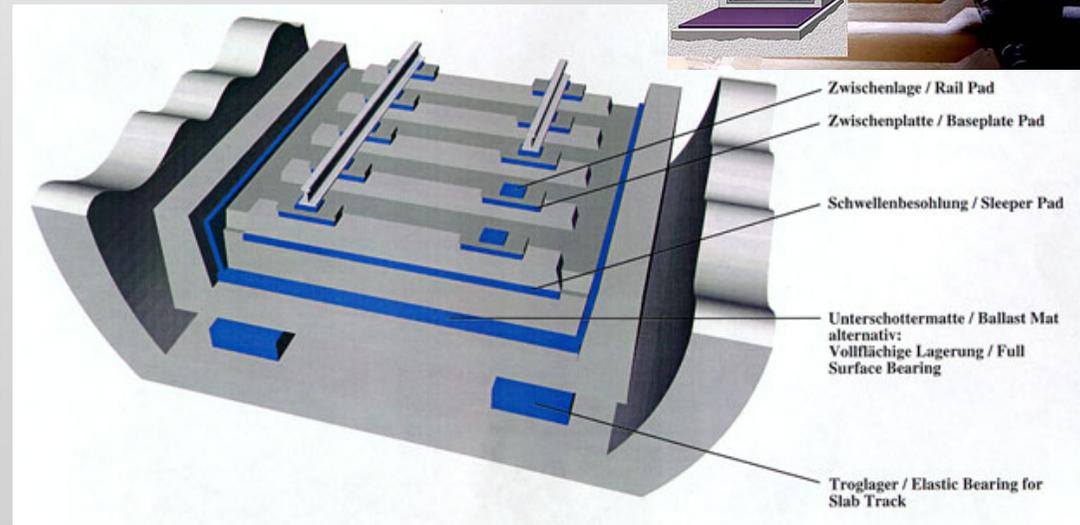
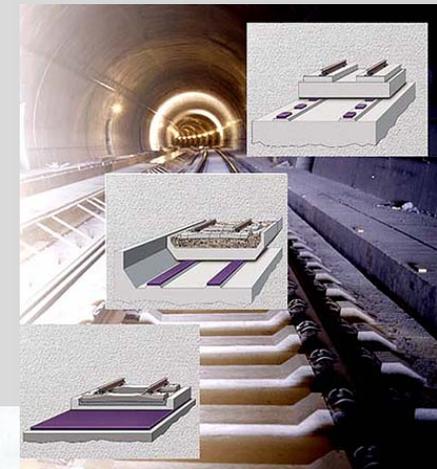
Derived from A. Kuznetsov diploma work  
at MAMI Moscow Technical University

# 7. Viscoelastic dampers for vibration and seismic protection

## 7.3. Shock and vibration absorbers in railway engineering



Shock absorbing resin-like material



# 7. Viscoelastic dampers for vibration and seismic protection

## 7.4. Vibration absorbers in bridge engineering



Resin-polymer vibration absorbers between span beams and columns (Bridge over the Rhone river, Villeurbanne, France)



# 7. Viscoelastic dampers for vibration and seismic protection

## 7.5. Shock and vibration absorbers in civil and industrial engineering

### 7.5.1. Dashpots (dampers) for seismic protection

#### 7.5.1.1. Dashpots by “Taylor Devices” and “Scott Forge”

##### A. Dashpot design

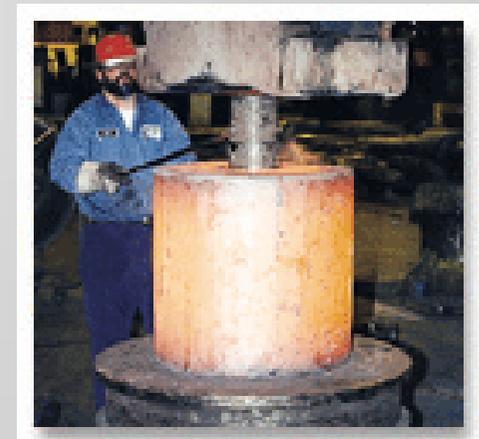
Dashpots by “Taylor Devices” (NY, USA)



Dashpots by “Scot Forge” (IL, USA)



Forged dampers were used in the Torre Mayor Building. Developers-Paul Reichmann; architects-Ziedler Roberts Partnership (both of Canada)



# 7. Viscoelastic dampers for vibration and seismic protection

## 7.5. Shock and vibration absorbers in civil and industrial engineering

### 7.5.1. Dashpots (dampers) for seismic protection

#### 7.5.1.1. Dashpots by “Taylor Devices” and “Scott Forge”

##### B. “Torre Mayor” building equipped with dashpots



The 55-story Torre Mayor (Mexico city), meaning "Big Tower," is the tallest building in Latin America

On January 21, 2003, Mexico city experienced a 7.6 magnitude earthquake, but occupants of the building did not suffer from this earthquake.

# 7. Viscoelastic dampers for vibration and seismic protection

## 7.5. Shock and vibration absorbers in civil and industrial engineering

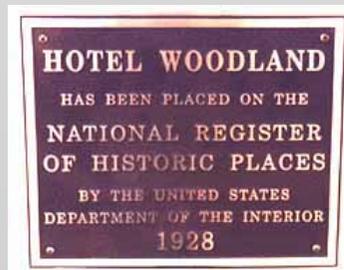
### 7.5.1. Dashpots (dampers) for seismic protection

#### 7.5.1.1. Dashpots by “Taylor Devices” and “Scott Forge”

##### C. Other structure equipped with dashpots

Hotel Woodland in Woodland California, USA

Notice, that dashpots are installed in the upper part of the frame!



# 7. Viscoelastic dampers for vibration and seismic protection

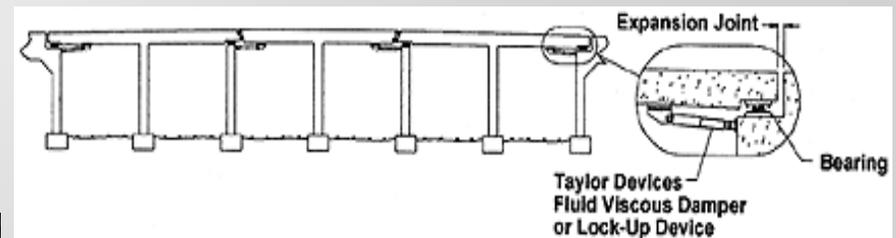
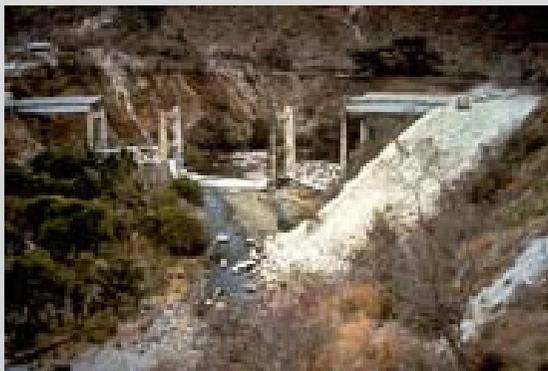
## 7.5. Shock and vibration absorbers in civil and industrial engineering

### 7.5.1. Dashpots (dampers) for seismic protection

#### 7.5.1.1. Dashpots by “Taylor Devices” and “Scott Forge”

##### D. Some applications in bridge constructing

Overall view of the collapse of the three central spans of the bridge at Agua Caliente (Guatemala) caused by the 1976 earthquake



# **7. Viscoelastic dampers for vibration and seismic protection**

## **7.5. Shock and vibration absorbers in civil and industrial engineering**

### **7.5.1. Dashpots (dampers) for seismic protection**

#### **7.5.1.1. Dashpots by “Taylor Devices” and “Scott Forge”**

##### **E. Ultimate capacity of the dashpots**

Capacity up to: 2,000,000 pounds (9072 KN)

Strokes of up to: 120 inches (3.048 m)

Temperatures: -40 ÷ +160 F (-40 ÷ +70 C)

35-year warranty

# 7. Viscoelastic dampers for vibration and seismic protection

## 7.5. Shock and vibration absorbers in civil and industrial engineering

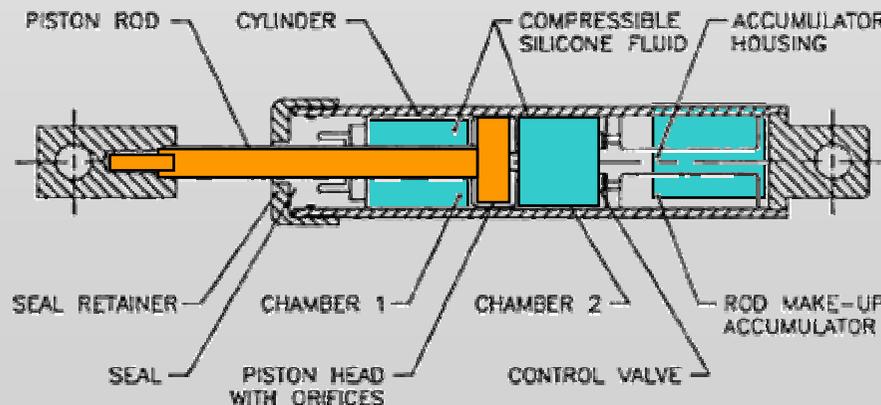
### 7.5.1. Dashpots (dampers) for seismic protection

#### 7.5.1.1. Dashpots by “Taylor Devices” and “Scott Forge”

#### F. Concluding remark

The “Taylor Devices” position their dashpots, as dampers, i.e. the systems composed of dashpots + springs (Kelvin elements).

#### Possible explanation



Valves in the piston!

# 7. Viscoelastic dampers for vibration and seismic protection

## 7.5. Shock and vibration absorbers in civil and industrial engineering

### 7.5.1. Dashpots (dampers) for seismic protection

#### 7.5.1.2. Dashpots by “Robinson Seismic Limited”



Two bearings, each weighing three-quarters of a tonne, are put through a seismic-simulation test rig



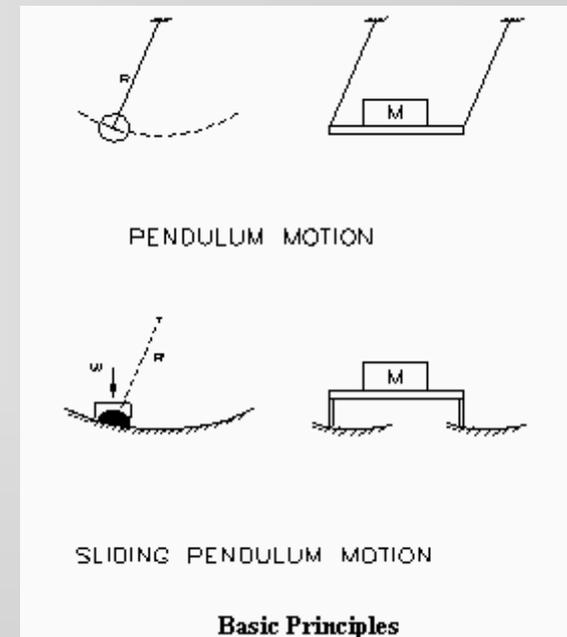
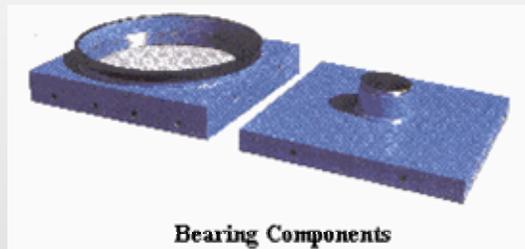
A bridge in Wellington,  
New Zealand with dashpots

# 7. Viscoelastic dampers for vibration and seismic protection

## 7.5. Shock and vibration absorbers in civil and industrial engineering

### 7.5.2. Friction Pendulum Seismic Isolation Bearings

#### 7.5.2.1. The main principle



By the “Earthquake Protection Systems, Inc.”, CA, USA

EPS Inc.

# 7. Viscoelastic dampers for vibration and seismic protection

## 7.5. Shock and vibration absorbers in civil and industrial engineering

### 7.5.2. Friction Pendulum Seismic Isolation Bearings

#### 7.5.2.2. Examples in civil engineering



San Francisco International  
Airport Terminal

by EPS Inc.



US Court of Appeals

# 7. Viscoelastic dampers for vibration and seismic protection

## 7.5. Shock and vibration absorbers in civil and industrial engineering

### 7.5.2. Friction Pendulum Seismic Isolation Bearings

#### 7.5.2.3. Example in bridge construction



Friction Pendulum Bearing in the “American River Bridge”  
at Lake Natoma in Folsom (CA)

by EPS Inc.

# 7. Viscoelastic dampers for vibration and seismic protection

## 7.5. Shock and vibration absorbers in civil and industrial engineering

### 7.5.3. Elastomeric dampers

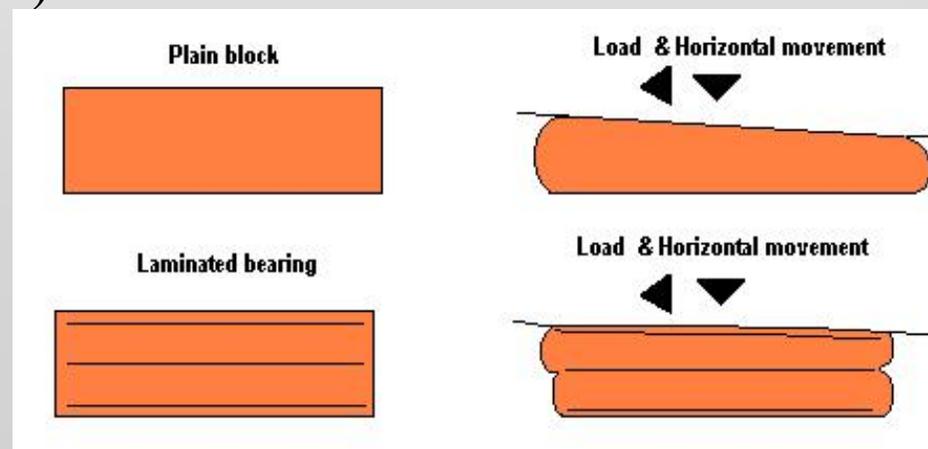
#### 7.5.3.1. Examples of design



Plane rubber  
(neoprene) mats



Laminated  
metal (lead)-rubber  
mats



# 7. Viscoelastic dampers for vibration and seismic protection

## 7.5. Shock and vibration absorbers in civil and industrial engineering

### 7.5.3. Elastomeric dampers

#### 7.5.3.2. Examples in civil and industrial engineering



Laminated Neoprene mats  
for seismic protection by  
AARP (UAE)



Solid neoprene mat for  
column bearing by Agom  
International srl (Italy)

# 7. Viscoelastic dampers for vibration and seismic protection

## 7.5. Shock and vibration absorbers in civil and industrial engineering

### 7.5.4. Software of analyzing vibrations

The software interface displays a 2D model of a two-story structure on the left. The top plot shows the El Centro Earthquake Acceleration (g) over time (t(s)), with a peak of 0.35. The bottom plot shows the Shear Force (kN) versus Displacement (m), with a peak of 330.0 kN. The right side of the interface contains several parameter tables and checkboxes.

**STRUCTURE PARAMETERS**

	First Story/Mode	Second Story/Mode
Floor Mass (tons) :	100	90
Stiffness (KN/m) :	5000	4000
Frequency (Hz) :	0.70	1.71
Damping (KN*s/m) :	30	10
Damping Ratio :	0.0107	0.0194

**STRUCTURE MODELS**

- Linear Model
- Nonlinear Damping Model
- Hysteretic (Bouc-Wen) Model
- Hysteretic (Bilinear) Model

**Excitation Parameters**

	First Story	Second Story
Involution Coefficient :	0.5	0.5
Yield Displacement (m) :	0.02	0.02
Post Yield Stiffness (KN/m) :	1000	1000

**RESPONSE PARAMETERS**

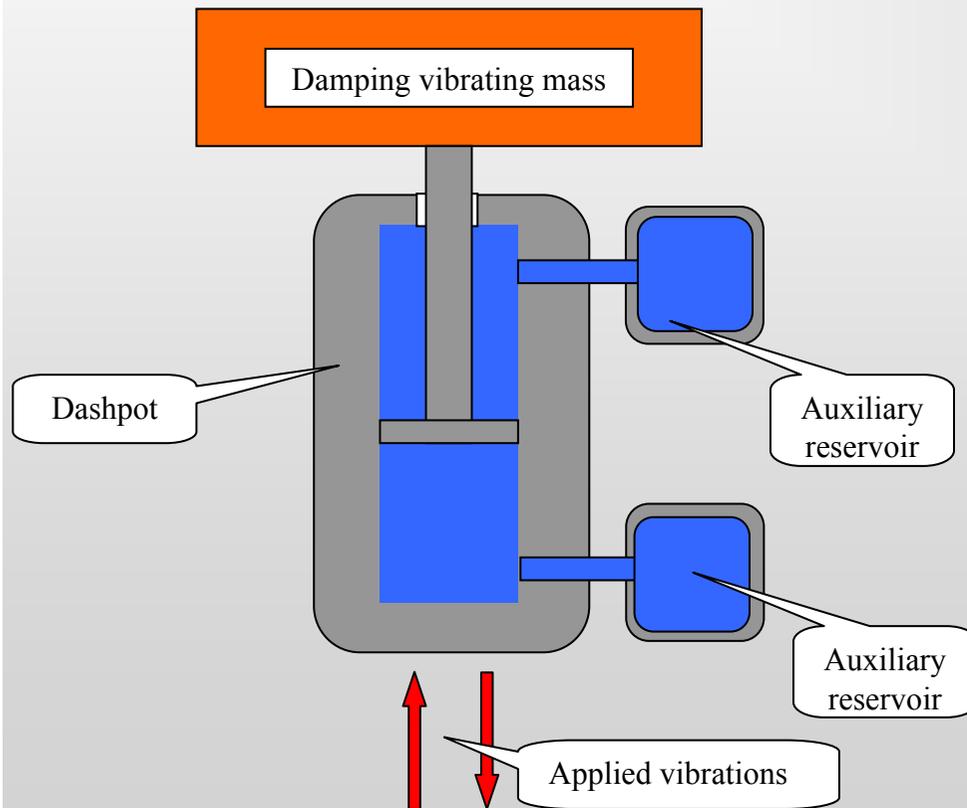
Maximum Amplitude (g) :	0.3495
Sinusoid Frequency (Hz) :	1.0
Response Window (sec) :	6

Buttons: Calculate, Animate, Reset Parameters

# 7. Viscoelastic dampers for vibration and seismic protection

## 7.6. Active shock absorbers

### 7.6.1. Vertical arrangement

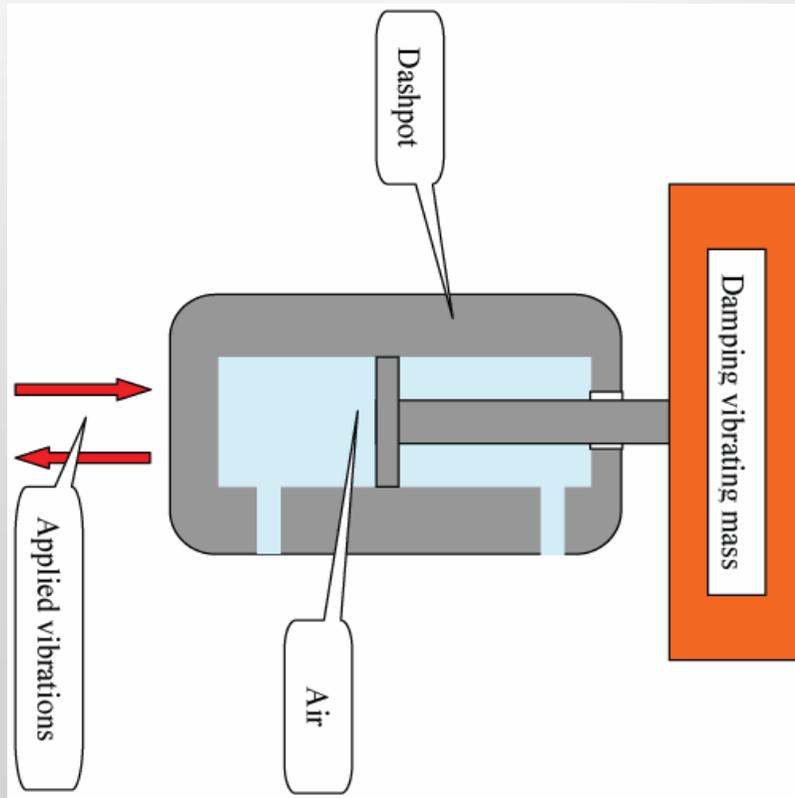


The main idea is to maintain the constant force acting on the upper mass to eliminate its vibrations, by using the so called “active” damping system

# 7. Viscoelastic dampers for vibration and seismic protection

## 7.6. Active shock absorbers

### 7.6.2. Horizontal arrangement

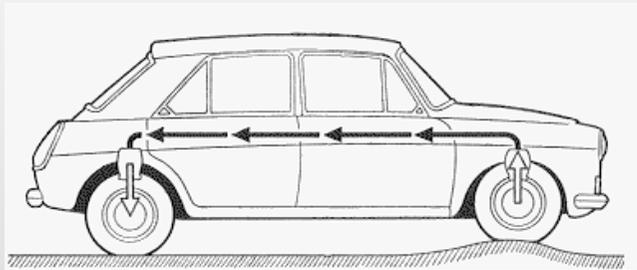


In the horizontal arrangement there is now need in maintaining constant liquid or gas pressure in the chambers. The strut will go freely.

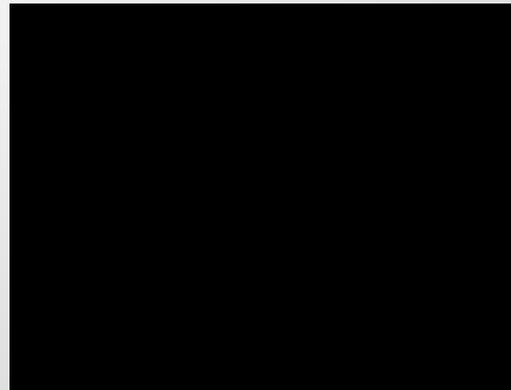
# 7. Viscoelastic dampers for vibration and seismic protection

## 7.6. Active shock absorbers

### 7.6.3. Analogy with the automotive industry



An example of “hydrolastic” suspension proposed for Mini by Dr. Alex Moulton



Example of electronically controlled (electromotor driven) suspension by Bose Co.



An example of horizontal displacement of dampers by “Racing for Holland” Co.

# 7. Viscoelastic dampers for vibration and seismic protection

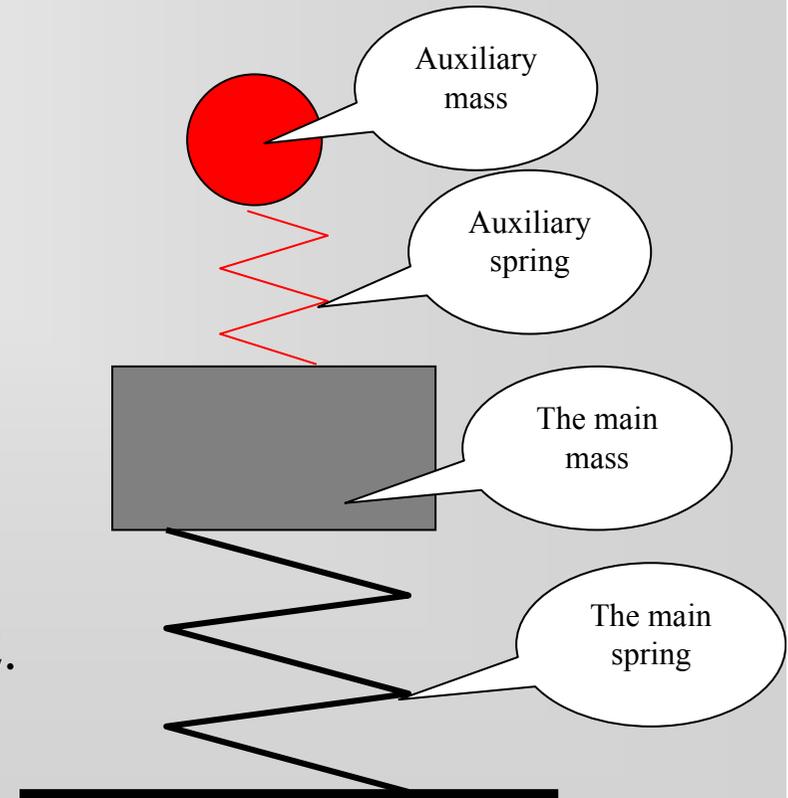
## 7.7. Another principle of vibration absorbing

### 7.7.1. The basic idea

Damping vibrations can also be achieved by applying to the vibrating mass some additional mass connected with the first one with a suitable spring element:

#### Remarks

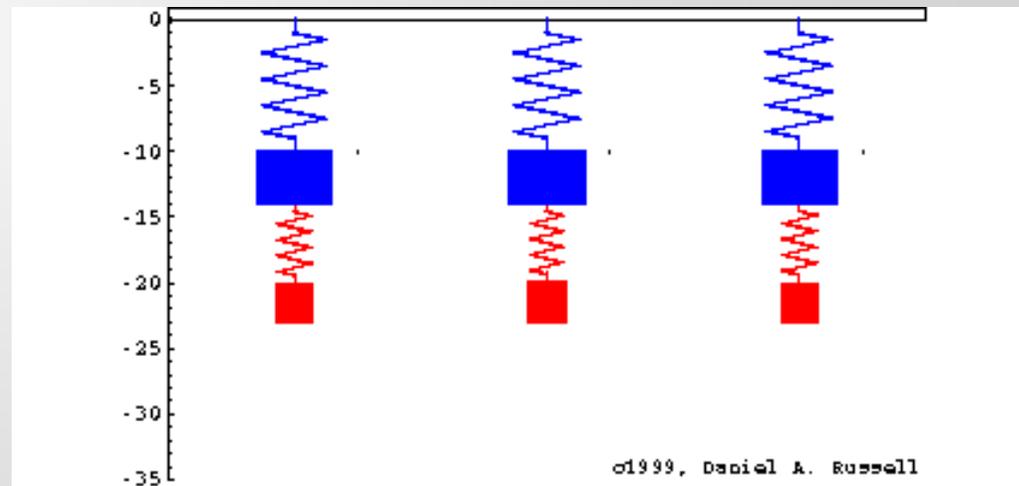
- Such systems are known as the 2 DOF.
- They lead to a coupled system of two ODE.
- This principle was suggested by J. Ormondroyd and J.P. Den Hartog in 1928



# 7. Viscoelastic dampers for vibration and seismic protection

## 7.7. Another principle of vibration absorbing

### 7.7.2. Visualization



#### Remarks

The oscillating force is applied to the main mass

The left system vibrates with a frequency  $0.67\omega_0$

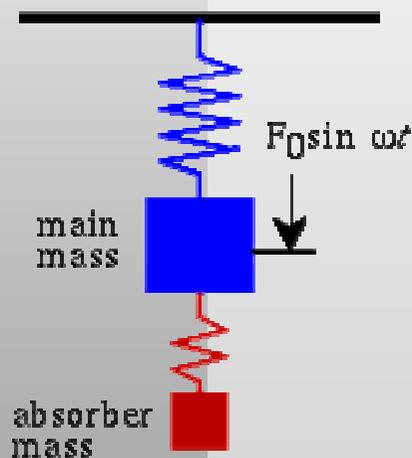
The middle system vibrates with the resonance frequency  $\omega_0$ !

The right system vibrates with the frequency  $1.3 \omega_0$ .

# 7. Viscoelastic dampers for vibration and seismic protection

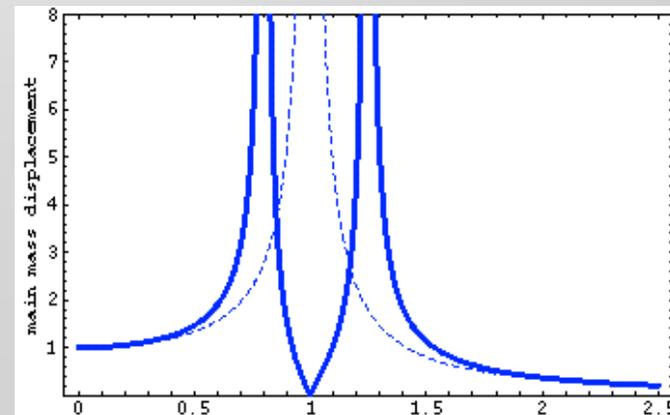
## 7.7. Another principle of vibration absorbing

### 7.7.3. The main equations



Coupled system of two ODE

$$\begin{cases} m_2 \ddot{x}_2 + k_2 x_2 - k_1 (x_1 - x_2) = F_0 \sin(\omega t) \\ m_1 \ddot{x}_1 + k_1 (x_1 - x_2) = 0 \end{cases}$$



# 8. Some hints on preventing appearing shock waves at building sites

## 8.1. Some observational data

Building site (Moscow, 2003)



Cracks in an apartment house



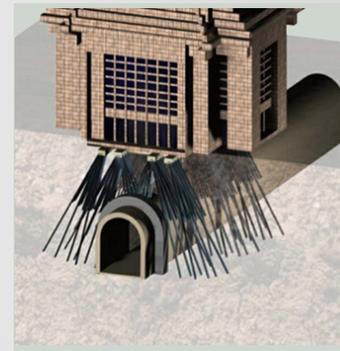
# 8. Some hints on preventing appearing shock waves at building sites

## 8.2. Techniques for eliminating shock waves at building sites

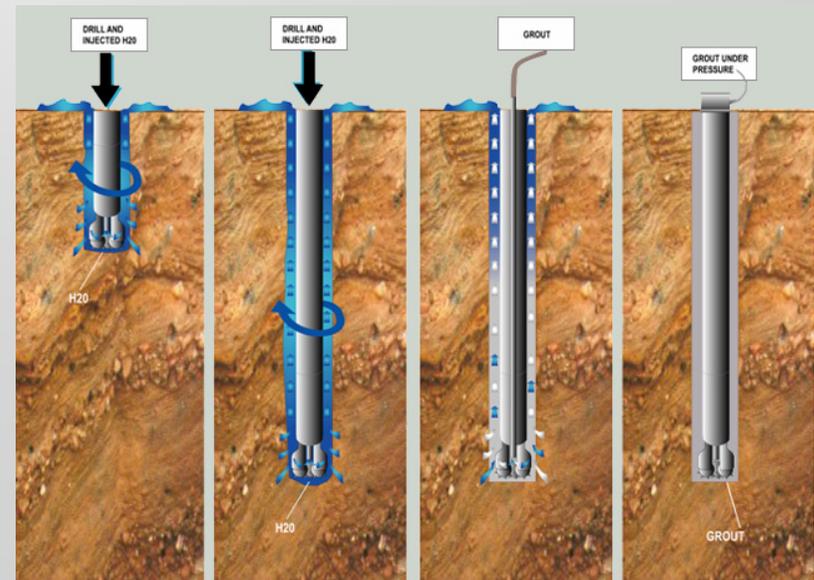
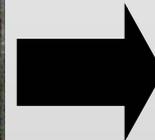
Diesel hummer  
for driving piles



Vibratory hammer  
for driving piles



Examples of the  
drilled piles  
(by "LayneGeo")



## Part IV. Review and conclusions

# 9. Concluding remarks

## 9.1. Brief summary

### 9.1.1. Seismic waves the main properties

Frequency range: 0.001 – 120Hz  
( 1 – 70Hz are the most dangerous)

Speed range: 100 – 6500 m/sec

Wavelength range: 2 – 6500 m

# 9. Concluding remarks

## 9.1. Brief summary

### 9.1.2. Seismic waves scales

Richter magnitude test scale:

Logarithmic scale (0 – 10)

The modified Mercalli intensity scale:

Scale of intensity in grades (I – XII)

# 9. Concluding remarks

## 9.1. Brief summary

### 9.1.3. Bulk waves the main properties

The main theorem:

1. For any direction in an arbitrary anisotropic medium, there are three bulk waves:
  - One (quasi) longitudinal and two (quasi) transverse waves
2. These waves can travel with (generally) different speeds
3. All bulk waves are not dispersing (wave speed does not depend upon frequency)

# 9. Concluding remarks

## 9.1. Brief summary

### 9.1.4. Surface acoustic waves classification

There are the following principle types of SAW:

1. **Rayleigh** waves (propagate in a halfspace)
2. **Stoneley** waves (propagate on an interface between two halfspaces)
3. **Love** waves (propagate in a layer and a halfspaces, have SH polarization)
4. **Lamb** waves (propagate in a layer)
5. **SH** waves (propagate in a layer, have H polarization)

# 9. Concluding remarks

## 9.1. Brief summary

### 9.1.5. The main principles of deterministic predicting analysis

1. Approximating using some analytical bases functions
2. Obtaining the desired extrapolation

#### **Remark**

Presumably, the best suited for the functions with the unknown periodicity and relatively short-time predictions are interpolating (Lagrange or Newton) polynomials, provided computations are done with the multiprecision arithmetic

# 9. Concluding remarks

## 9.1. Brief summary

### 9.1.6. The main principles of seismic wave protection

1. Modifying surface layers
  - a. Creating “rough” surfaces
  - b. Modifying physical properties of the surface layer
2. Creating seismic barriers
  - a. Barriers reflecting seismic waves
  - b. Barriers scattering wave energy
3. Installing dampers
  - a. Discrete dampers composed of a dashpot and a spring
  - b. Continuous type viscoelastic dampers (pads)

# 9. Concluding remarks

## 9.2. Directions for further studies

- 9.2.1. Theoretical studies of surface acoustic waves (analytical methods);
- 9.2.2. Interaction of bulk or surface waves with barriers (mainly FEM);
- 9.2.3. Scattering of elastic waves by inclusions and creating wave scattering barriers (mainly FEM);
- 9.2.4. Vibro- and seismic damper engineering (mainly ODE);
- 9.2.5. Methods and algorithms for predicting (mainly numerical methods).

# **9. Concluding remarks**

## **9.3. The concluding note**

# 9. Concluding remarks

## 9.4. Recommended literature

### 9.4.1. Earth structure

Van der Pluijm Ben A. and Marshak Stephen,  
*Earth structure*,  
McGraw-Hill, 2002, ISBN: 0697172341

Pollard David D., Fletcher Raymond C.,  
*Fundamentals of Structural Geology*,  
Cambridge Univ. Press, 2005, ISBN: 10- 052183927

# 9. Concluding remarks

## 9.4. Recommended literature

### 9.4.2. Experimental methods in geophysics

D.J. Rockhill, D.J. White, and M.D. Bolton,  
*Ground vibrations due to piling operations*,  
BGA Int. Conf. on Foundations, 2003, Dundee, Scotland, 743-756

Stephen E. Prensky,  
*A Survey of Recent Developments and Emerging Technology in Well Logging and  
Rock Characterization*,  
The Log Analyst, 1994, v. 35, N.2, p.15-45, N.5, p.78-84 (extensive bibliography )

# 9. Concluding remarks

## 9.4. Recommended literature

### 9.4.3. Theory of anisotropic elasticity

Rand Omri and Rovenski Vladimir Y.

*Analytical Methods in Anisotropic Elasticity with Symbolic Computational Tools*,  
Birkhäuser, 2005, 451 p., ISBN-10: 0-8176-4272-2

Ting T.C.T.,

*Anisotropic Elasticity: Theory and Applications*,  
Oxford Univ. Press, 1996, ISBN-13: 9780195074475

Lekhnitskii S. G.,

*Theory of Elasticity of an Anisotropic Body*,  
Holden-Day, San Francisco, 1963.

# 9. Concluding remarks

## 9.4. Recommended literature

### 9.4.4. Theory of acoustic elastic waves

Auld B.A.,

*Acoustic Fields and Waves in Solids,*

2nd edition, Krieger Pub Co, 1990, ISBN: 0894644904

Kaufman A.A. and Levshin A.L.,

*Acoustic and Elastic Wave Fields in Geophysics,*

Parts I - III, Elsevier, 2000, ISBN: 0-444-50336-6

Kennett B. L. N.,

*Seismic Wave Propagation in Stratified Media,*

Cambridge University Press; Reprint edition, 1985, ISBN: 0521312191

# 9. Concluding remarks

## 9.4. Recommended literature

### 9.4.5. Engineering methods in protecting from seismic waves

Hansbo S.,  
*Foundation Engineering*,  
Elsevier, 1994, ISBN-13: 978-0-444-88549-4

*Earthquake Proof Design and Active Faults*,  
Editor Y. Kanaori, Elsevier, 1997, ISBN-13: 978-0-444-82562-9

Mahtab M.A. and Grasso P.,  
*Geomechanics Principles in the Design of Tunnels and Caverns in Rocks*,  
Elsevier, 1992, ISBN-13: 978-0-444-88308-7