Acoustic methods in Rock Mechanics

Sergey V. Kuznetsov

Institute for Problems in Mechanics
Russian Academy of Sciences

e-mail: kuzn-sergey@yandex.ru

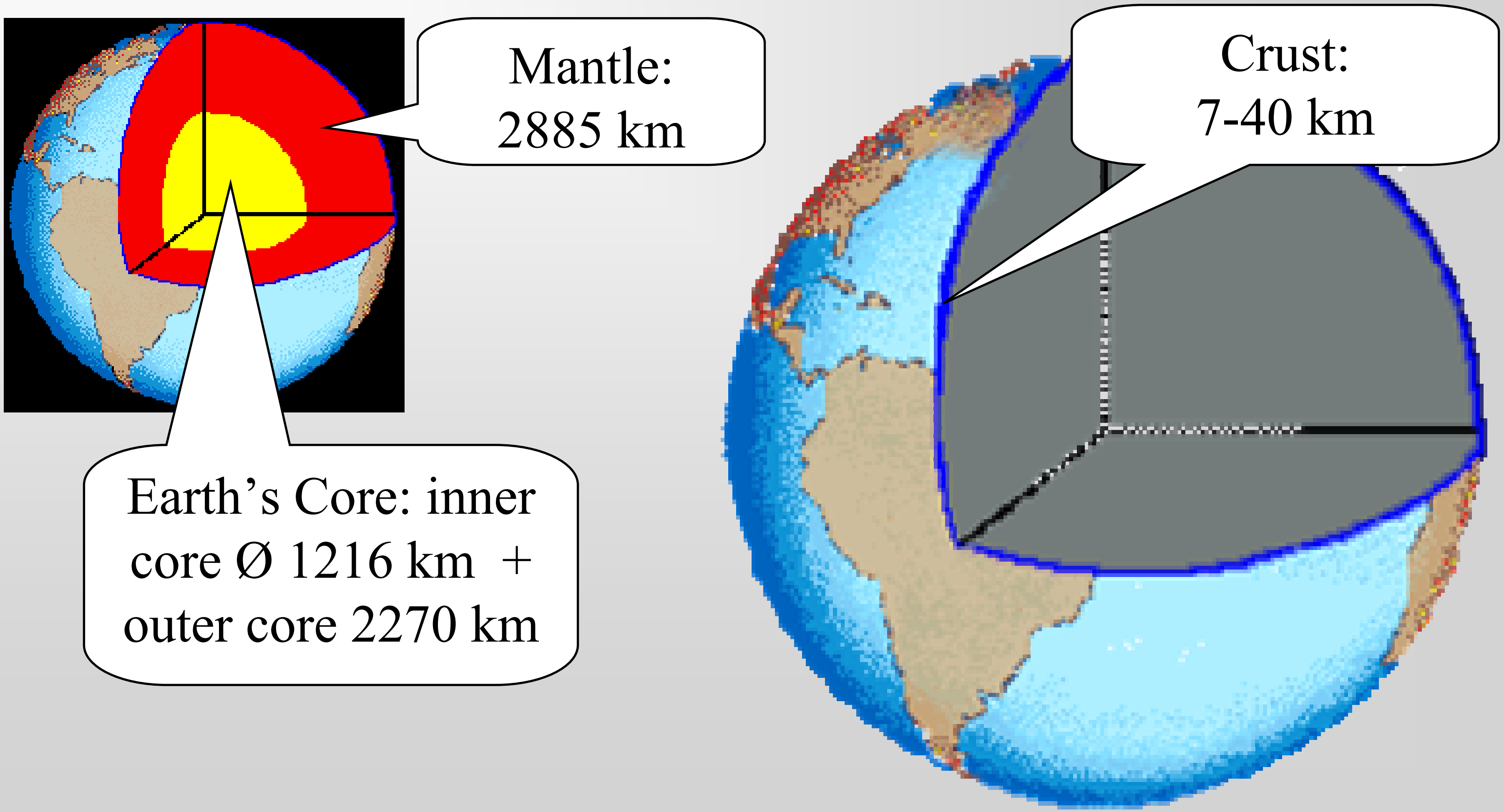
Part I. General considerations

1. Introduction

1.1. Position of acoustic methods in rock mechanics

1.1.1. Earth's structure

1.1.1.1. Earth's schematic structure



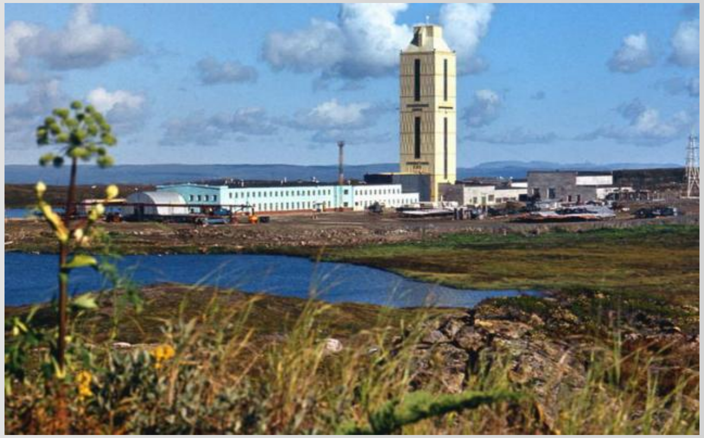
1. Introduction

1.1. Position of acoustic methods in rock mechanics

1.1.1. Earth’s structure

1.1.1.2. The non-acoustical method

Dates	Place	Depth (m) achieved
XIII Century	China	1300
1930-1935 1940-1955	Germany USA	5000 7000
1970-1992	Russia	12262 the deepest borehole in the world



1. Introduction

1.1. Position of acoustic methods in rock mechanics

1.1.2. Specimens for laboratory analyses

Kern samples

Bore-drill head

1. Introduction

1.1. Position of acoustic methods in rock mechanics

1.1.3. Some devices used for static measurements

INSTRON Testing Machine

Suitable for both static and dynamic loadings

Cycle loading up to 20Hz, suitable for the fatigue strength limit determination

Suitable for both creep and relaxation tests

1. Introduction

1.1. Position of acoustic methods in rock mechanics

1.1.4. Experimental determination of mechanical properties of a kern

Kern: Borehole No. 000-01bis

Site: Villeurbanne Region, Central France

Coordinates: $45^0 45'27''$(NL); $4^0 47'07''$(Greenwich value)

Time: 16h 35min 27sec (GMT)

Temperature: $+15.2^0$C

Barometric pressure: 987mm mercury

1. Introduction

1.2. Some experimental data concerning propagation of acoustic waves in rocks and soils

1.2.1. Basic definitions

A wave propagating in a given material is called **subsonic** (**supersonic**), if its speed is below (greater) than the corresponding speed of the longitudinal wave propagating in the same direction

A wave is called **hyposonic** (**ultrasonic**), if its oscillations in time are below (greater) than the audible frequencies: 20 – 20 000 Hz

1. Introduction

1.2. Some experimental data concerning propagation of acoustic waves in rocks and soils

1.2.2. Frequency range

Wave nature	Frequency range
Seismic waves of natural origin	0.001 - 50Hz Most dangerous: 1-50Hz
Waves of artificial nature	10 - 120Hz Most dangerous: 10-70Hz

1. Introduction

1.2. Some experimental data concerning propagation of acoustic waves in rocks and soils

1.2.3. Speed range for longitudinal waves

Material	**Speed m/sec**
air	330 - 360
Bullet in the air	~800
soil	200 - 800
sand	100 - 1000
water	1430 - 1590
slate (shale)	2000 - 5000
limestone	3000 - 6000
granite	4500 - 6500

1. Introduction

1.2. Some experimental data concerning propagation of acoustic waves in rocks and soils

1.2.4. Wavelength ranges

1.2.4.1. Wavelength range for seismic longitudinal waves

$$l = c/\omega,$$

where

l is wavelength

c is speed

ω is frequency

Wavelengths for seismic longitudinal waves from 1 to 50 Hz

Material	**Wavelength m**
soil	40 - 800
sand	2 - 1000
water	29 - 1590
slate (shale)	40 - 5000
limestone	60 - 6000
granite	90 - 6500

1. Introduction

1.2. Some experimental data concerning propagation of acoustic waves in rocks and soils

1.2.4. Wavelength ranges

1.2.4.2. Remark for wavelength of seismic waves

$$l = c/\omega,$$

where

l is wavelength

c is speed

ω is frequency

As was pointed out previously there can be seismic waves propagating with very low frequencies (0.01-1Hz), for such waves the corresponding wavelength can be sufficiently larger, than pointed in the previous table. Thus, for granites the wavelength of longitudinal waves can have up to 650 km.

1. Introduction

1.2. Some experimental data concerning propagation of acoustic waves in rocks and soils

1.2.4. Wavelength ranges

1.2.4.3. Wavelength range for artificial longitudinal waves

$$l = c/\omega,$$

where

l is wavelength

c is speed

ω is frequency

Wavelengths for artificial longitudinal waves from 10 to 70 Hz

Material	**Wavelength m**
soil	28 - 80
sand	1 - 100
water	20 - 159
slate (shale)	28 - 500
limestone	42 - 600
granite	63 - 650

1. Introduction

1.3. Seismic waves scales

1.3.1. Richter magnitude scale

Charles Richter (1935) arbitrarily chose a magnitude 0 event to be an earthquake that would show a maximum combined horizontal displacement of 1 micrometer on a seismogram recorded using a Wood-Anderson torsion:

$$M_L = \log_{10} A(mm) + (Distance\ correction\ factor)$$

Remark

According to Richter scale, earthquakes of magnitude

<3	are not felt	(frequency is ~1000 per day)
5 - 6	can cause damage	(~800 per year)
7 - 8	serious damage	(~18 per year)
8 - 9	severe damage	(~1 per year)
>9	extreme damage	(~1 per 20 years)

1. Introduction

1.3. Seismic waves scales

1.3.2. Mercalli *intensity* scale

The ***Modified Mercalli Intensity Scale*** (originated to seismologist Giuseppe Mercalli, 1902) is commonly used by assigning numbers **I – XII** according to severity of the earthquake effects, so there may be many the modified Mercalli intensity values for each earthquake, depending upon distance of the epicenter.

Remark

According to the Mercalli modified intensity scale:

I. People do not feel any Earth movement.
II. A few people might notice movement.
III. Many people indoors feel movement.
IV. Most people indoors feel movement.
…………………..
XII. Almost everything is destroyed.

1. Introduction

1.3. Seismic waves scales

1.3.3. The greatest earthquake

According to Richter scale, the greatest recorded earthquake occurred on 22d May, 1960 in Chili (The Great Chilean Earthquake or Valdivia Earthquake).

This earthquake was measured 9.5 by Richter scale.

Remark

The earthquake caused localized tsunami that hit the Chilean coast severely, with waves up to 25 meters high. The main tsunami ran through the Pacific Ocean and hit Hawaii, where waves as high as 10.7 meters high were recoded

2. The main targets of acoustical studies

2.1. Analyses of physical and geometrical properties of rocks and soils

2.2. Observation of possible internal faults and cracks

2.3. Observation, analyses, and monitoring of the underground water and oil reservoirs

2.4. Analyses of seismic (natural) and artificial vibrations

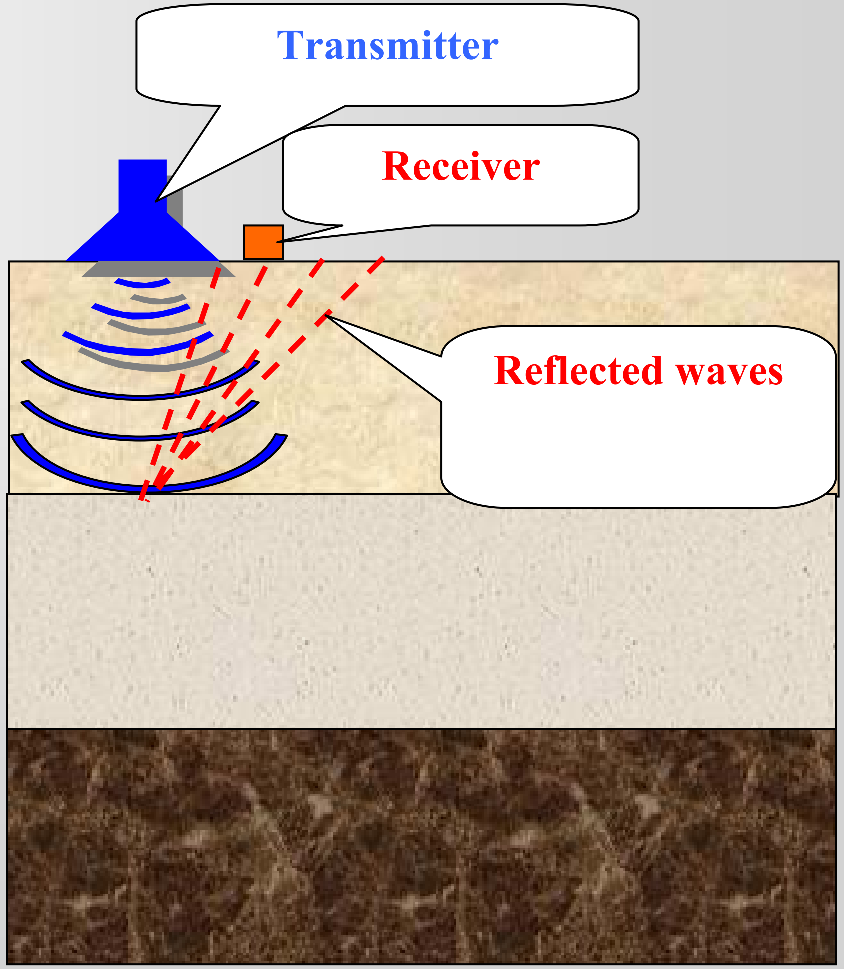
2.5. Prediction of earthquakes

2.6. Developing methods and technologies for surface wave protection

3. Experimental techniques in acoustical studies

3.1. Methods based on reflected acoustical waves

3.1.1. Basic principle



3. Experimental techniques in acoustical studies

3.1. Methods based on reflected acoustical waves

3.1.2. History of the reflected acoustic wave measurements

Oklahoma is the birthplace of the reflection seismic technique of oil exploration. This geophysical method records reflected seismic waves as they travel through the earth, helping to find oil-bearing formations. It has been responsible for discovery of many of the world's largest oil and gas fields, containing billions of barrels of oil and trillions of cubic feet of natural gas.

Pioneering research and development was led by Dr. J. C. Karcher, an Oklahoma physicist. The Arbuckle Mountains of Oklahoma were selected for a pilot survey of the technique and equipment, because an entire geologic section from the basal Permian to the basement mass of granite is exposed here. This survey followed limited testing in June, 1921 in the outskirts of Oklahoma City

The world's first reflection seismograph geologic section was measured on August 9, 1921 along Vines Branch, a few miles north of Dougherty (Oklahoma region)

3. Experimental techniques in acoustical studies

3.1. Methods based on reflected acoustical waves

3.1.3. Emitters (transmitters) of acoustic waves

Method	Gained frequencies	Comments
Explosions	10-150Hz	Not used now
Falling objects (rams)	10-70Hz	Widely used
Impulse gas detonators	10-100Hz	Used
Vibro-platforms	10-30Hz	Widely used
Existing sources of vibration	10-120Hz	Used

3. Experimental techniques in acoustical studies

3.1. Methods based on reflected acoustical waves

3.1.4. Registering devices for reflected waves

1. Seismographs, velocitometers, accelerometers

Seismometer
(KS2000)

2. Analog-digital converter

ADC 4-channel
(Dataq DI 148U)

3. Experimental techniques in acoustical studies

3.1. Methods based on reflected acoustical waves

3.1.5. Remark on the reflected acoustical wave method

Advantages:

+ Easy to receive and analyze experimental data
+ Relatively good accuracy for the first layer
+ Possibility to analyze layered media

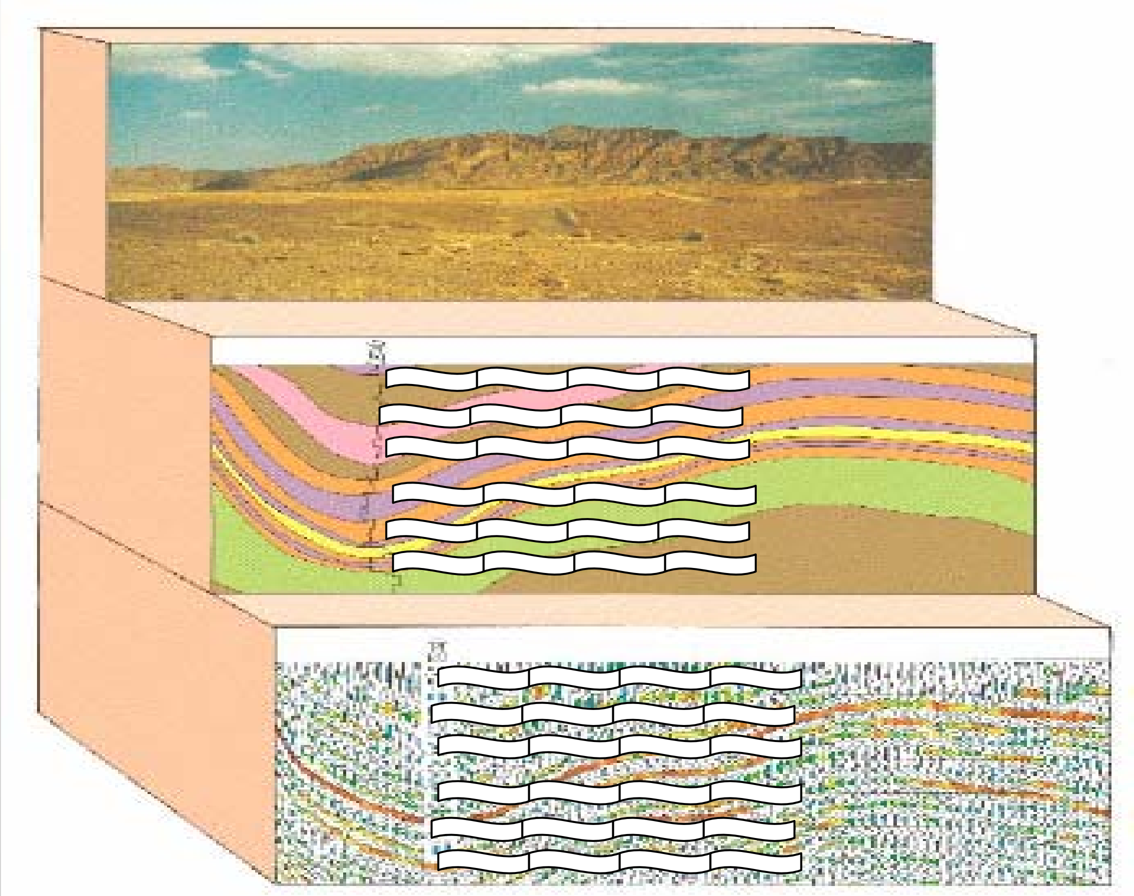
Disadvantages:

- Generally, low accuracy for layered media
- Impossibility to analyze media with “hard” internal layers

3. Experimental techniques in acoustical studies

3.2. Methods based on surface acoustic waves

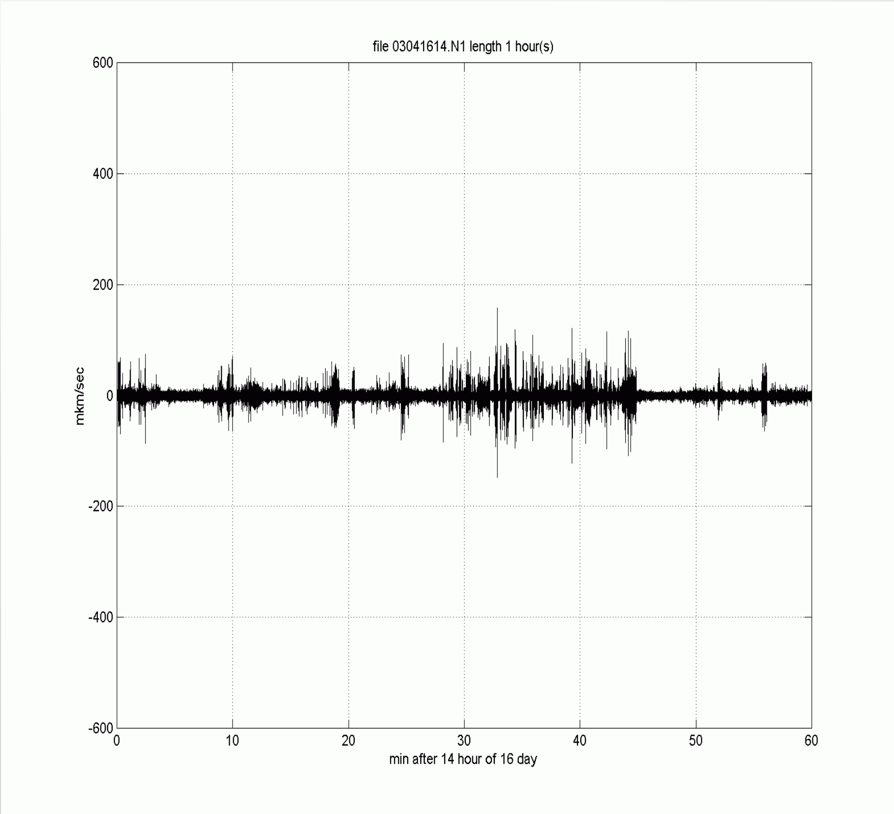
3.2.1. Basic principle



3. Experimental techniques in acoustical studies

3.2. Methods based on acoustical surface waves

3.2.2. Use of natural seismic background



An example of a seismogram containing natural seismic background, due to propagating surface acoustic waves (Moscow, 2003)

3. Experimental techniques in acoustical studies

3.2. Methods based on acoustical surface waves

3.2.3. Use of artificially generated surface acoustic waves

Method	Gained frequencies	Comments
Explosions	10-150Hz	Not used now
Falling objects (rams)	10-70Hz	Widely used
Impulse gas detonators	10-100Hz	Used
Vibro-platforms	10-30Hz	Widely used
Existing sources of vibration	10-120Hz	Used

3. Experimental techniques in acoustical studies

3.2. Methods based on acoustical surface waves

3.2.4. Methods of registration

The same, as were used for methods based on the reflected waves

Seismograph, velocitometer, accelerometer

Analog-digital converter

3. Experimental techniques in acoustical studies

3.2. Methods based on acoustical surface waves

3.2.5. Remark on the surface acoustical wave method

Advantages:

+ Possibility to use the natural seismic background
+ Possibility to analyze multilayered media
+ Possibility to analyze media with “hard” internal layers

Disadvantages:

- More complicated than the reflected wave method

Part II. Theoretical analyses

4. Theoretical methods in acoustical studies

4.1. Basic notations

4.1.1. The main operators of mathematical physics

$$\nabla \equiv \left(\frac{\partial}{\partial x_1}; \dots; \frac{\partial}{\partial x_n} \right) \quad \Rightarrow$$

$$\nabla f\left(x_1;\dots;x_n\right) = \left(\frac{\partial f}{\partial x_1}; \dots; \frac{\partial f}{\partial x_n} \right) = \left(f_{,1}; \dots; f_{,n} \right)$$

$$\operatorname{div} \equiv \left(\nabla \cdot\right) \quad \Rightarrow$$

$$\operatorname{div} \mathbf{g}\left(x_1;\dots;x_n\right) = \left(\frac{\partial g_1}{\partial x_1} + \dots + \frac{\partial g_n}{\partial x_n} \right) =$$

$$\left(g_{1,1} + \dots + g_{n,n}\right) = \sum_{k=1}^{n} g_{k,k} = g_{k,k}$$

4. Theoretical methods in acoustical studies

4.1. Basic notations

4.1.2. Notion of stress and strain tensors

$$\sigma \equiv \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix}$$

$$\varepsilon \equiv \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{13} & \varepsilon_{23} & \varepsilon_{33} \end{pmatrix}$$

4. Theoretical methods in acoustical studies

4.1. Basic notations

4.1.3. Basic equations of elastodynamics

Cauchy relations:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right)$$

or

$$\varepsilon = \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^t\right)$$

Cauchy equations of motion:

$$\sum_{j=1}^{3} \frac{\partial \sigma_{ij}}{\partial x_j} + \rho f_i = \rho \ddot{u}_i, \qquad i = 1,2,3$$

or

$$\operatorname{div} \boldsymbol{\sigma} + \rho \mathbf{f} = \rho \ddot{\mathbf{u}}$$

4. Theoretical methods in acoustical studies

4.2. Introduction to the anisotropic elasticity theory

4.2.1. The generalized Hook's law written in a 3-dimensional formalism

$$\sigma_{ij} = \sum_{m=1}^{3}\sum_{n=1}^{3} C^{ijmn}\varepsilon_{mn} = \underbrace{C^{ijmn}\varepsilon_{mn}}_{\text{Einstein's notatin}}$$

or

$$\sigma = \mathbf{C}\cdot\cdot\varepsilon$$

Remark

Tensor **C** is symmetric with respect to the outer pairs of indexes, and has 21 maximum independent components.

4. Theoretical methods in acoustical studies

4.2. Introduction to the anisotropic elasticity theory

4.2.2. Six-dimensional formalism in anisotropic elasticity

4.2.2.1. Six-dimensional stress and strain vectors

$$\vec{\sigma} = \begin{pmatrix} \sigma_1 = \sigma_{11} \\ \sigma_2 = \sigma_{22} \\ \sigma_3 = \sigma_{33} \\ \sigma_4 = \sigma_{23} \\ \sigma_5 = \sigma_{13} \\ \sigma_6 = \sigma_{12} \end{pmatrix} \qquad \vec{\varepsilon} = \begin{pmatrix} \varepsilon_1 = \varepsilon_{11} \\ \varepsilon_2 = \varepsilon_{22} \\ \varepsilon_3 = \varepsilon_{33} \\ \varepsilon_4 = \varepsilon_{23} \\ \varepsilon_5 = \varepsilon_{13} \\ \varepsilon_6 = \varepsilon_{12} \end{pmatrix}$$

4. Theoretical methods in acoustical studies

4.2. Introduction to the anisotropic elasticity theory

4.2.2. Six-dimensional formalism in anisotropic elasticity

4.2.2.2. The generalized Hook's law in a 6-dim formalism

$$\vec{\sigma} = \hat{\mathbf{C}}_{6\times 6} \cdot \vec{\varepsilon}$$

Remark

$$\hat{\mathbf{C}}_{6\times 6} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ & & C_{33} & C_{34} & C_{35} & C_{36} \\ & & & C_{44} & C_{45} & C_{46} \\ & & & & C_{55} & C_{56} \\ & & & & & C_{66} \end{pmatrix}$$

4. Theoretical methods in acoustical studies

4.2. Introduction to the anisotropic elasticity theory

4.2.3. Basic kinds of elastic anisotropy

4.2.3.1. Triclinic medium

$$\hat{\mathbf{C}}_{6\times 6} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ & & C_{33} & C_{34} & C_{35} & C_{36} \\ & & & C_{44} & C_{45} & C_{46} \\ & & & & C_{55} & C_{55} \\ & & & & & C_{66} \end{pmatrix}$$

Remark

21 independent elastic constants,
no planes of elastic symmetry

4. Theoretical methods in acoustical studies

4.2. Introduction to the anisotropic elasticity theory

4.2.3. Basic kinds of elastic anisotropy

4.2.3.2. Monoclinic medium

$$\hat{\mathbf{C}}_{6\times 6} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ & C_{22} & C_{23} & 0 & 0 & C_{26} \\ & & C_{33} & 0 & 0 & C_{36} \\ & & & C_{44} & C_{45} & 0 \\ & & & & C_{55} & 0 \\ & & & & & C_{66} \end{pmatrix}$$

Remark

13 independent elastic constants,
one plane of elastic symmetry (plane < 12 >),
example: ore mineral "Colemanite"

4. Theoretical methods in acoustical studies

4.2. Introduction to the anisotropic elasticity theory

4.2.3. Basic kinds of elastic anisotropy

4.2.3.3. Orthotropic (orthorhombic) medium

$$\hat{\mathbf{C}}_{6\times 6} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ & C_{22} & C_{23} & 0 & 0 & 0 \\ & & C_{33} & 0 & 0 & 0 \\ & & & C_{44} & 0 & 0 \\ & & & & C_{55} & 0 \\ & & & & & C_{66} \end{pmatrix}$$

Remark

9 independent elastic constants,
three plane of elastic symmetry,
example: ore mineral “Parkerite
”

4. Theoretical methods in acoustical studies

4.2. Introduction to the anisotropic elasticity theory

4.2.3. Basic kinds of elastic anisotropy

4.2.3.4. Cubic symmetry

$$\hat{\mathbf{C}}_{6\times 6} = \begin{pmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ & C_{11} & C_{12} & 0 & 0 & 0 \\ & & C_{11} & 0 & 0 & 0 \\ & & & C_{44} & 0 & 0 \\ & & & & C_{44} & 0 \\ & & & & & C_{44} \end{pmatrix}$$

Remark

3 independent elastic constants,
three planes of elastic symmetry,
example: ore mineral “Pyrite” (FeS_2)

4. Theoretical methods in acoustical studies

4.2. Introduction to the anisotropic elasticity theory

4.2.3. Basic kinds of elastic anisotropy

4.2.3.5. Transversely isotropic medium

$$\hat{\mathbf{C}}_{6\times 6} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ & C_{11} & C_{13} & 0 & 0 & 0 \\ & & C_{33} & 0 & 0 & 0 \\ & & & C_{44} & 0 & 0 \\ & & & & C_{44} & 0 \\ & & & & & (C_{11}-C_{12})/2 \end{pmatrix}$$

Remark

5 independent elastic constants,
three planes of elastic symmetry,
a lot of composite materials are transversely isotropic

4. Theoretical methods in acoustical studies

4.2. Introduction to the anisotropic elasticity theory

4.2.3. Basic kinds of elastic anisotropy

4.2.3.6. Isotropic medium

$$\hat{\mathbf{C}}_{6\times 6} = \begin{pmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ & C_{11} & C_{12} & 0 & 0 & 0 \\ & & C_{12} & 0 & 0 & 0 \\ & & & (C_{11}-C_{12})/2 & 0 & 0 \\ & & & & (C_{11}-C_{12})/2 & 0 \\ & & & & & (C_{11}-C_{12})/2 \end{pmatrix}$$

Remark

2 independent elastic constants,
infinite number of planes of elastic symmetry,
a lot of polycrystalline and amorphous materials

4. Theoretical methods in acoustical studies

4.2. Introduction to the anisotropic elasticity theory

4.2.4. Inequalities imposed on the elasticity tensor

4.2.4.1. Arbitrary elastic anisotropy

Inequality ensuring positive (specific) deformation energy:

$$A \equiv \frac{1}{2}\varepsilon \cdot\cdot \sigma > 0 \quad \Rightarrow$$

$$\Rightarrow \quad \frac{1}{2}\varepsilon \cdot\cdot \mathbf{C} \cdot\cdot \varepsilon = \frac{1}{2}\vec{\varepsilon} \cdot \hat{\mathbf{C}}_{6\times 6} \cdot \vec{\varepsilon} > 0 \qquad \forall \vec{\varepsilon}$$

Where

$$\hat{\mathbf{C}}_{6\times 6} = \mathbf{Q}^{t}{}_{6\times 6} \cdot \mathbf{D}_{6\times 6} \cdot \mathbf{Q}_{6\times 6}$$

Remark

$\mathbf{D}_{6\times 6}$ is a diagonal matrix

And, condition of positive deformation energy requires all the diagonal elements of matrix $\mathbf{D}_{6x6}$ to be positive

4. Theoretical methods in acoustical studies

4.2. Introduction to the anisotropic elasticity theory

4.2.4. Inequalities imposed on the elasticity tensor

4.2.4.2. Isotropic medium

$$\hat{\mathbf{C}}_{6\times 6} = \begin{pmatrix} \lambda+2\mu & \lambda & \lambda & & & \\ \lambda & \lambda+2\mu & \lambda & & & \\ \lambda & \lambda & \lambda+2\mu & & & \\ & & & 2\mu & & \\ & & & & 2\mu & \\ & & & & & 2\mu \end{pmatrix} =$$

$$= \hat{\mathbf{Q}}_{6\times 6}{}^{t} \cdot \begin{pmatrix} 3\lambda+2\mu & & & & & \\ & 2\mu & & & & \\ & & 2\mu & & & \\ & & & 2\mu & & \\ & & & & 2\mu & \\ & & & & & 2\mu \end{pmatrix} \cdot \hat{\mathbf{Q}}_{6\times 6}$$

$$\Rightarrow \quad 3\lambda+2\mu > 0, \quad \mu > 0$$

$$\Downarrow$$

$$E > 0$$

$$\frac{1}{2} > \nu > -1$$

4. Theoretical methods in acoustical studies

4.2. Introduction to the anisotropic elasticity theory

4.2.5. Remark on experimental determination of elastic constants

A problem

How many “stones” are needed to prepare samples for defining elastic anisotropy and the corresponding elastic constants?

4. Theoretical methods in acoustical studies

4.3. Bulk wave propagation in anisotropic elastic media

4.3.1. Introduction

4.3.1.1. Basic definitions

Definitions

A wave is a periodic or quasi periodic movement in time and space.

Wave front is the geometrical set of points vibrating with the same phase.

Types of waves according to the wave front

Spherical waves

Cylindrical waves

Waves with a plane wave front

4. Theoretical methods in acoustical studies

4.3. Bulk wave propagation in anisotropic elastic media

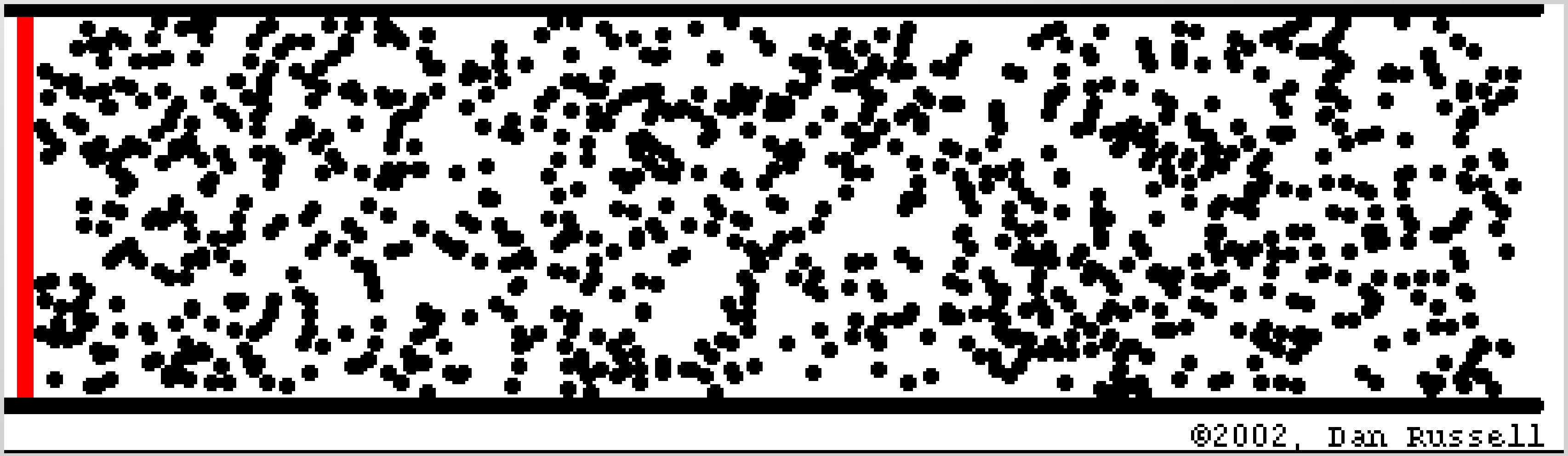
4.3.1. Introduction

4.3.1.2. Remark on no mass transfer

Remark

At wave motion no mass transfer occurs.

This is applied to all the (linear) theories of acoustic waves.



4. Theoretical methods in acoustical studies

4.3. Bulk wave propagation in anisotropic elastic media

4.3.2. The main equations for bulk waves

4.3.2.1. Representation for a wave with the plane wave front

$$\mathbf{u}(\mathbf{x},t) = \mathbf{m}\, e^{ir(\mathbf{n}\cdot\mathbf{x} - ct)}$$

Where

$\mathbf{u}$ is a displacement field

$\mathbf{x}$ is a space variable

t is time

$\mathbf{m}$ is the amplitude (polarization) of the wave

r is the wave number ($r = 2\pi / l$, or $r = \omega / c$)

$\mathbf{n}$ is direction of propagation ($\mathbf{n}$ is the unit vector)

c is the phase speed

4. Theoretical methods in acoustical studies

4.3. Bulk wave propagation in anisotropic elastic media

4.3.2. The main equations for bulk waves

4.3.2.2. Acoustic tensor

Substituting representation for the plane wave front into equation of motion yields

$$ir\left(\mathbf{n}\cdot\mathbf{C}\cdot\mathbf{n}-\rho c^{2}\mathbf{I}\right)\cdot\mathbf{m}=0$$

Definition for the acoustic tensor

$$\mathbf{A}(\mathbf{n})\equiv\mathbf{n}\cdot\mathbf{C}\cdot\mathbf{n}$$

Remark

The acoustic tensor can be constructed for any direction **n**, and it is symmetric and positive definite for any kind of elastic anisotropy

4. Theoretical methods in acoustical studies

4.3. Bulk wave propagation in anisotropic elastic media

4.3.2. The main equations for bulk waves

4.3.2.3. Christoffel equations

$$\left(\mathbf{A}(\mathbf{n}) - \rho c^2 \mathbf{I}\right) \cdot \mathbf{m} = 0 \quad \Longrightarrow \quad \det\left(\mathbf{A}(\mathbf{n}) - \rho c^2 \mathbf{I}\right) = 0$$

$$\Downarrow$$

$$\mathbf{Q}^t \cdot \left(\mathbf{D}_{\mathbf{A}(\mathbf{n})} - \rho c^2 \mathbf{I}\right) \cdot \mathbf{Q} = 0$$

Remark

$$\mathbf{D}_{\mathbf{A}(\mathbf{n})} = \begin{pmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{pmatrix} \quad \Longrightarrow \quad c_k = \sqrt{\frac{\lambda_k}{\rho}}, \quad k = 1, 2, 3$$

4. Theoretical methods in acoustical studies

4.3. Bulk wave propagation in anisotropic elastic media

4.3.2. The main equations for bulk waves

4.3.2.4. Polarization

$$\mathbf{A}(\mathbf{n}) = \lambda_1 \mathbf{m}_1 \otimes \mathbf{m}_1 + \lambda_2 \mathbf{m}_2 \otimes \mathbf{m}_2 + \lambda_3 \mathbf{m}_3 \otimes \mathbf{m}_3$$

where

$\mathbf{m}_k, \quad k = 1, 2, 3$ Are the mutually orthogonal and normal eigenvectors of the acoustical tensor

Corollary

Polarization vectors corresponding to different eigenvalues of the acoustic tensor are just its eigenvectors

4. Theoretical methods in acoustical studies

4.3. Bulk wave propagation in anisotropic elastic media

4.3.2. The main equations for bulk waves

4.3.2.5. Classification of bulk waves according to polarization

Definitions

A wave is called **longitudinal** (or **P**-wave)), if polarization **m** coincides with the direction of propagation **n**

A wave is called **transverse** (or **S**-wave), if polarization **m** is orthogonal to the direction of propagation **n**

A wave is called **quasi longitudinal**, if the scalar product $\mathbf{m} \cdot \mathbf{n} > 0$.

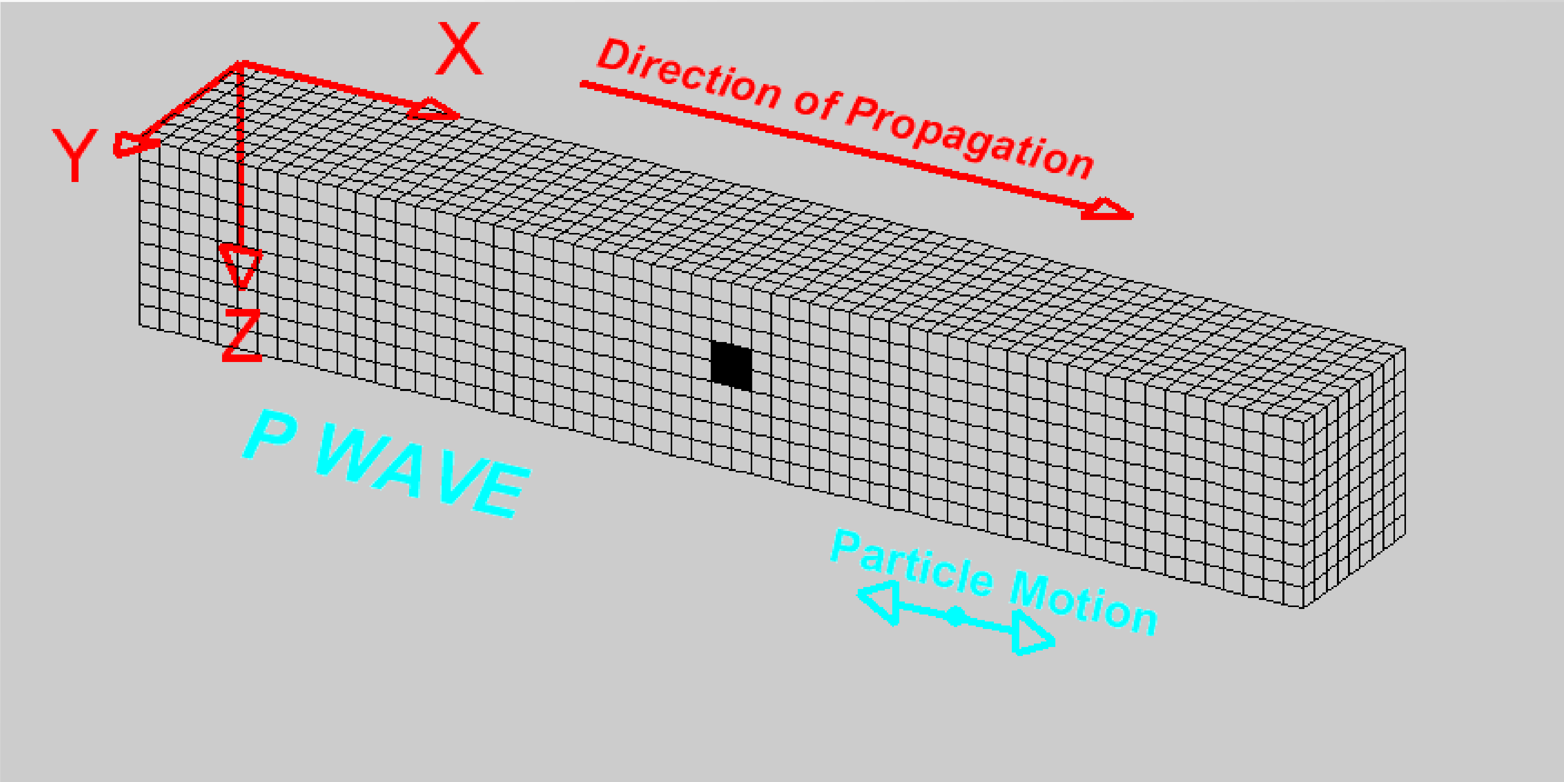
A wave is called **quasi transverse**, if the scalar product $\mathbf{m} \cdot \mathbf{n} < 0$.

4. Theoretical methods in acoustical studies

4.3. Bulk wave propagation in anisotropic elastic media

4.3.2. The main equations for bulk waves

4.3.2.6. Visual representation for polarization of P- waves

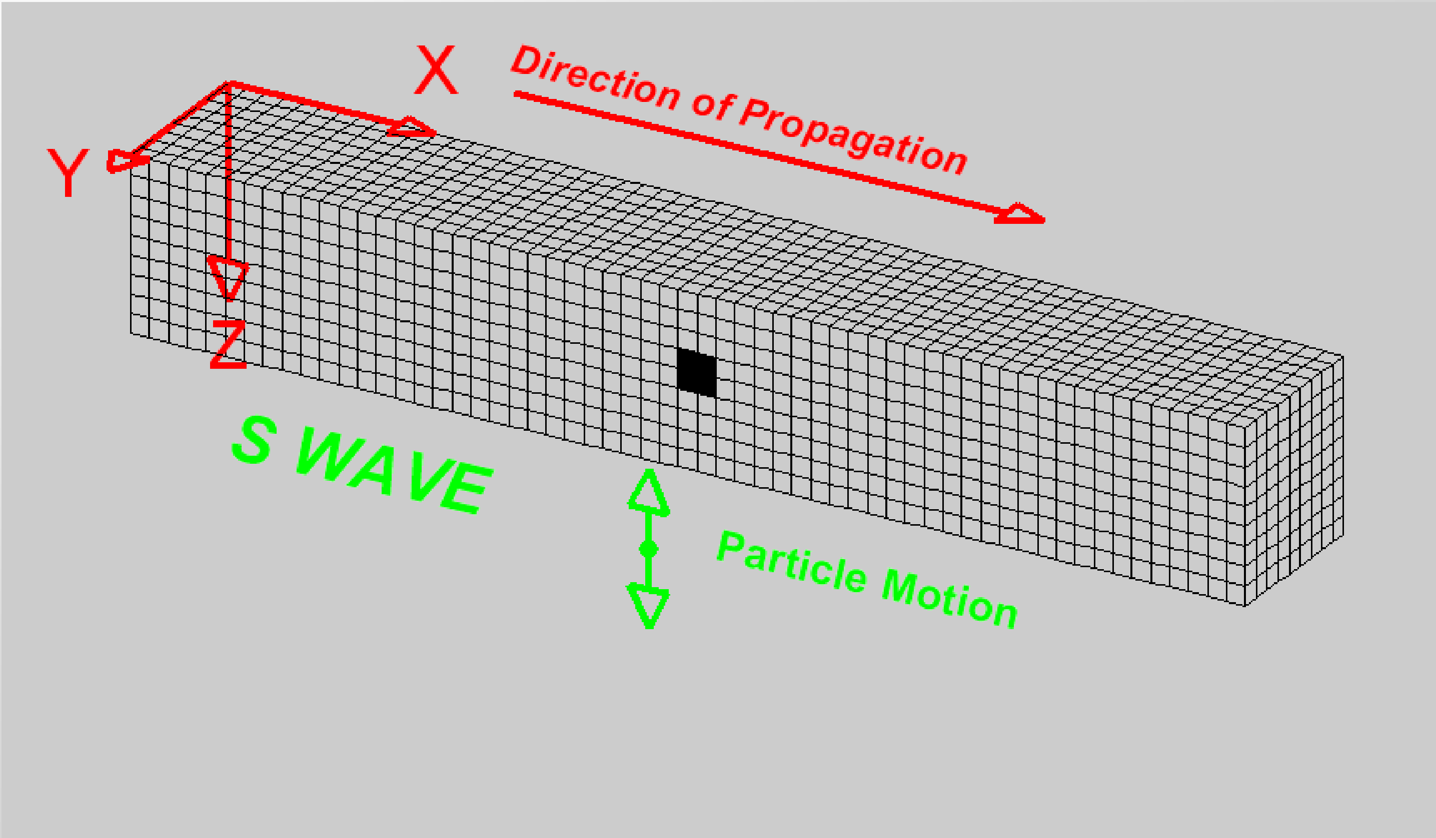


4. Theoretical methods in acoustical studies

4.3. Bulk wave propagation in anisotropic elastic media

4.3.2. The main equations for bulk waves

4.3.2.7. Visual representation for polarization of S- waves



4. Theoretical methods in acoustical studies

4.3. Bulk wave propagation in anisotropic elastic media

4.3.2. The main equations for bulk waves

4.3.2.8. Remarks on speeds of bulk waves

Remark 1

In most cases speed of the longitudinal (or quasi longitudinal) wave exceeds speeds of the transverse waves (or quasi transverse), that is why longitudinal waves are called P-waves (Primary waves)

But, there are some exceptions: TeO_2, in which one of transverse waves travels faster than the longitudinal

Remark 2

In isotropic materials at any admissible Lamé's constants $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$, speed of transverse waves is (strictly) lower than speed of the longitudinal wave.

4. Theoretical methods in acoustical studies

4.3. Bulk wave propagation in anisotropic elastic media

4.3.3. The main theorems for bulk waves

4.3.3.1. Theorem on existence of three bulk waves

Theorem

For any direction of propagation **n** of an arbitrary anisotropic medium:

(i) there exist three bulk waves, propagating with (generally) different phase speeds; and

(ii) having mutually orthogonal polarization vectors;

(iii) speeds of all bulk waves do not depend upon frequency

Remark

While speed of propagation of these bulk waves can coincide, their polarization vectors differ (and they must be mutually orthogonal).

4.3. Bulk wave propagation in anisotropic elastic media

4.3.3. The main theorems for bulk waves

4.3.3.2. Theorem on existence of the acoustic axes

Definition

An axis in an anisotropic medium is called "acoustic", if along it the longitudinal bulk wave can propagate.

Theorem

For any anisotropic medium, there exist at least three different acoustic axes.

4. Theoretical methods in acoustical studies

4.3. Bulk wave propagation in anisotropic elastic media

4.3.4. Specific energy of bulk waves

4.3.4.1. Kinetic energy

Definition

$$E_{kin} = \frac{1}{2}\rho\left|\dot{\mathbf{u}}\right|^2$$

Corollary

Substituting representation for the bulk wave into definition for kinetic energy, yields

$$E_{kin} = \frac{1}{2}\rho\left\langle\dot{\mathbf{u}}\cdot\overline{\dot{\mathbf{u}}}\right\rangle^2 = \frac{1}{2}\rho r^2 c^2 \cancel{\left\langle\mathbf{m}\cdot\overline{\mathbf{m}}\right\rangle}^{=1} = \frac{1}{2}\rho\omega^2$$

Remark

Thus, kinetic energy does not depend upon the phase speed c.

4. Theoretical methods in acoustical studies

4.3. Bulk wave propagation in anisotropic elastic media

4.3.4. Specific energy of bulk waves

4.3.4.2. Elastic (potential) energy

Definition

$$E_{elast} = \frac{1}{2}\varepsilon \cdot\cdot \overline{\sigma}$$

Corollary

Substituting representation for the bulk wave into definition for elastic energy, yields

$$E_{elast} = \frac{1}{2}\varepsilon \cdot\cdot \mathbf{C} \cdot\cdot \overline{\varepsilon} = \frac{1}{2}\nabla\mathbf{u} \cdot\cdot \mathbf{C} \cdot\cdot \overline{\nabla\mathbf{u}} = \frac{1}{2}r^2\,\mathbf{m}\cdot\overbrace{\left(\mathbf{n}\cdot\mathbf{C}\cdot\mathbf{n}\right)}^{\mathbf{A}(\mathbf{n})}\cdot\overline{\mathbf{m}} =$$

$$= \frac{1}{2}r^2\,\mathbf{m}\cdot\mathbf{A}(\mathbf{n})\cdot\overline{\mathbf{m}}$$

4.3. Bulk wave propagation in anisotropic elastic media

4.3.4. Specific energy of bulk waves

4.3.4.3. Theorem on elastic energy for bulk waves

Theorem

$$E_{elast} = E_{kin}$$

Proof

$$E_{elast} = \frac{1}{2} r^2\, \mathbf{m}\cdot \mathbf{A}(\mathbf{n})\cdot \mathbf{m}$$

but

$$\left(\mathbf{A}(\mathbf{n}) - \rho c^2 \mathbf{I}\right)\cdot \mathbf{m} = 0$$

now

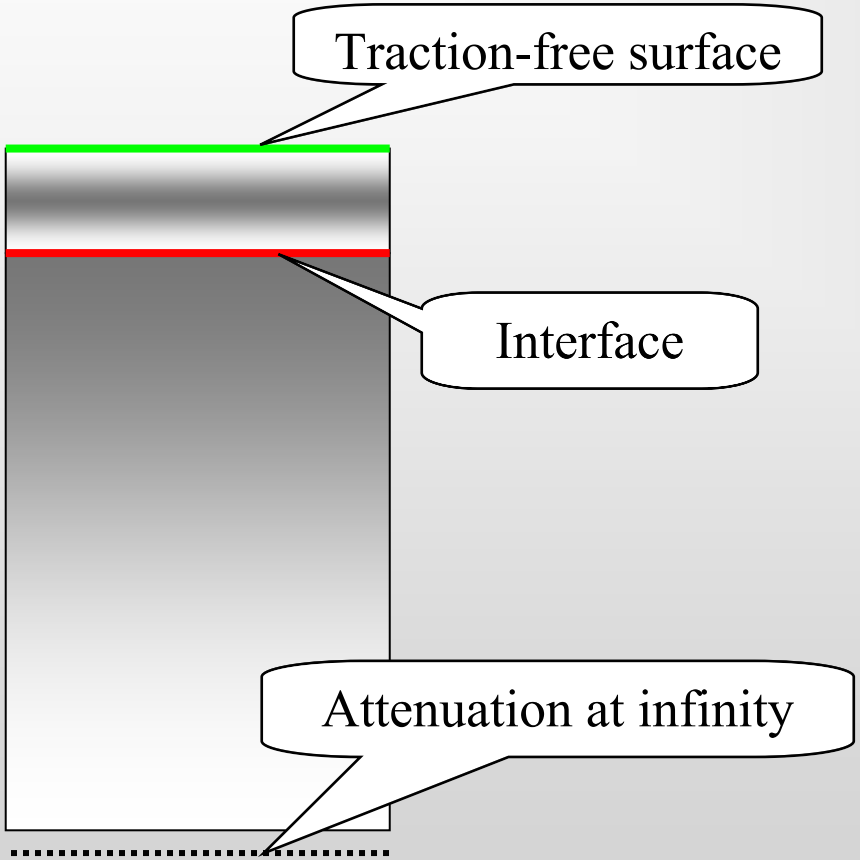
$$\mathbf{A}(\mathbf{n})\cdot \mathbf{m} = \rho c^2 \mathbf{m}$$

$$\mathbf{m}\cdot \mathbf{A}(\mathbf{n})\cdot \mathbf{m} = \rho c^2 \cancel{\langle \mathbf{m}\cdot \mathbf{m}\rangle}^{=1}$$

4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.1. Boundary, interface, and Sommerfield's conditions



Traction-free surface:

$$\mathbf{t}_\nu \equiv \nu \cdot \mathbf{C} \cdot\cdot \nabla \mathbf{u}\Big|_{x'=x_0} = 0$$

Interface:

$$\mathbf{t}_{\nu_+}{}^{upper\,layer} = \mathbf{t}_{\nu_-}{}^{lower\,layer\,(substrate)}$$

$$\mathbf{u}^{upper\,layer} = \mathbf{u}^{lower\,layer\,(substrate)}$$

Sommerfield's attenuation:

$$\left|\nabla \mathbf{u}(x')\right| = o(\left|x'\right|^{-1}), \qquad \left|x'\right| \to \infty$$

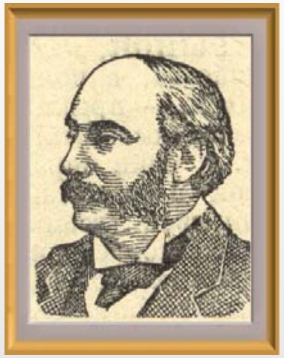
4. Theoretical methods in acoustical studies

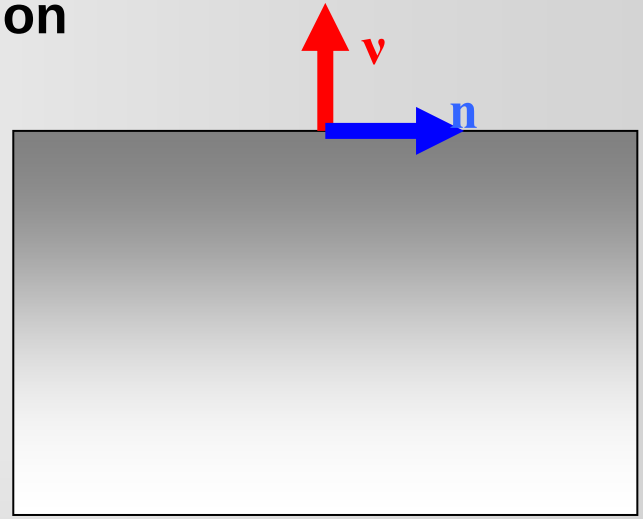
4.4. Surface acoustic waves

4.4.2. Classification of surface acoustic waves

4.4.2.1. Rayleigh waves

A. Basic definition





Definition

Rayleigh surface wave means attenuating with depth elastic wave with a plane wave front propagating on a traction-free boundary of a half-space (substrate).

Remark

The pioneering Rayleigh work, where these waves for the first time were described, appeared in 1885.

4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.2. Classification of surface acoustic waves

4.4.2.1. Rayleigh waves

B. The main properties

The main theorem

For a long time it was supposed that there can be anisotropic materials (may be artificial), that possesses specific directions along which Rayleigh waves cannot propagate. These hypothetical directions where called “forbidden”.

But, in 1973-1976 Barnett and Lothe proved a theorem **on existence** of Rayleigh wave for any anisotropic materials and any directions.

Another property

Rayleigh waves do not possess a dispersion (dependence of frequency on speed).

4. Theoretical methods in acoustical studies

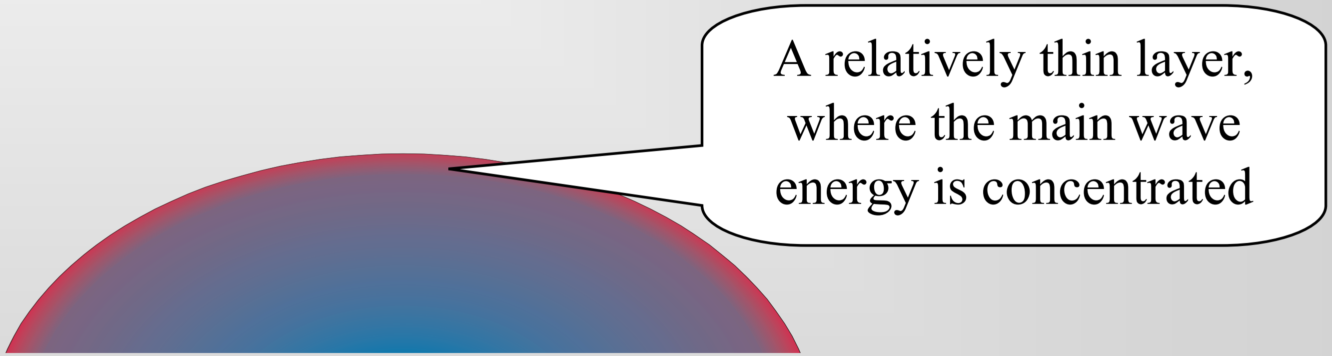
4.4. Surface acoustic waves

4.4.2. Classification of surface acoustic waves

4.4.2.1. Rayleigh waves

C. Role of Rayleigh waves in transmitting energy

These waves play a very important role in transmitting the seismic energy and causing the catastrophic destructions due to the seismic activity.



Remark

The amplitude of oscillations of Rayleigh wave attenuates exponentially with depth

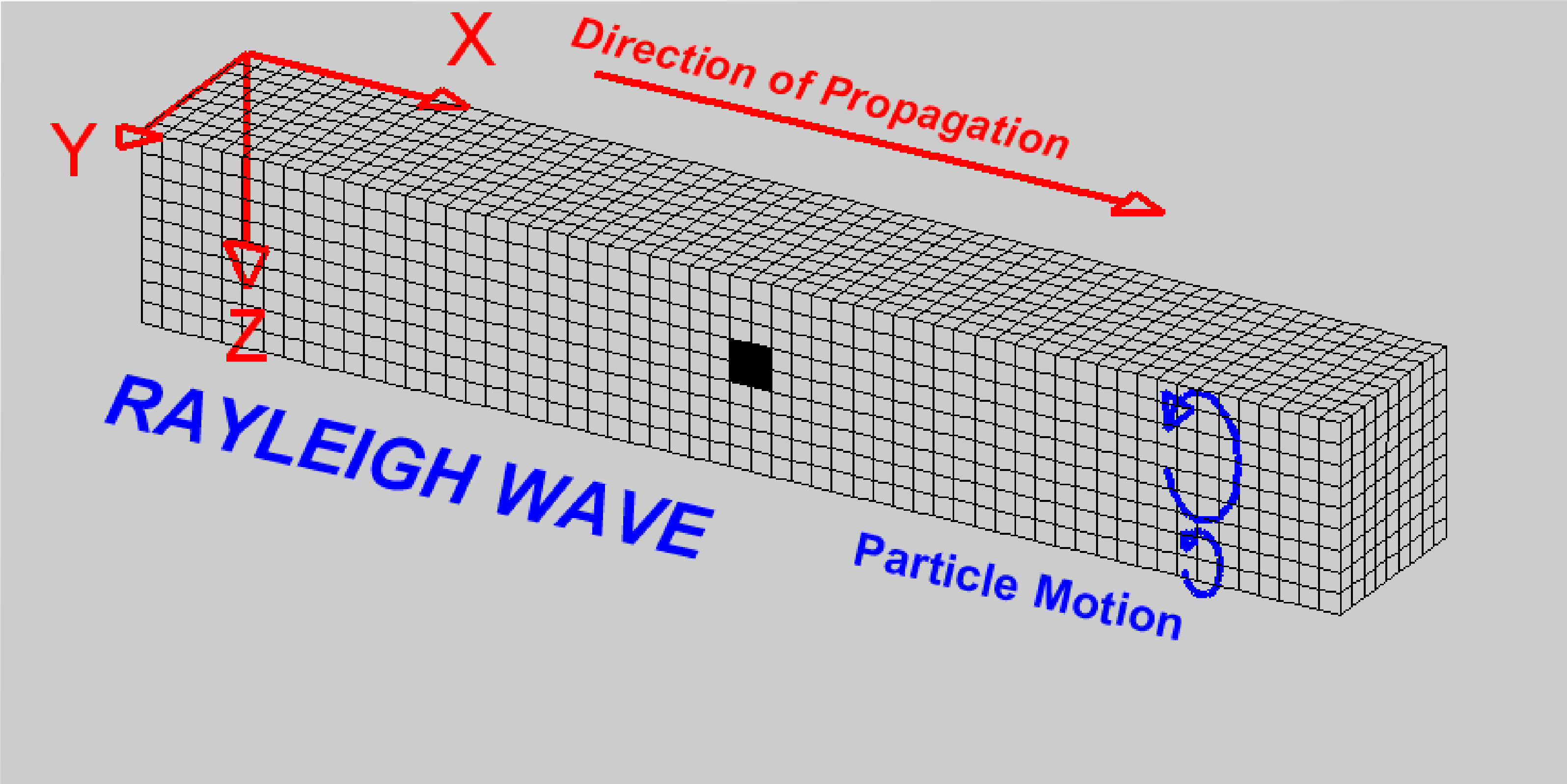
4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.2. Classification of surface acoustic waves

4.4.2.1. Rayleigh waves

D. Visualization of Rayleigh wave propagation



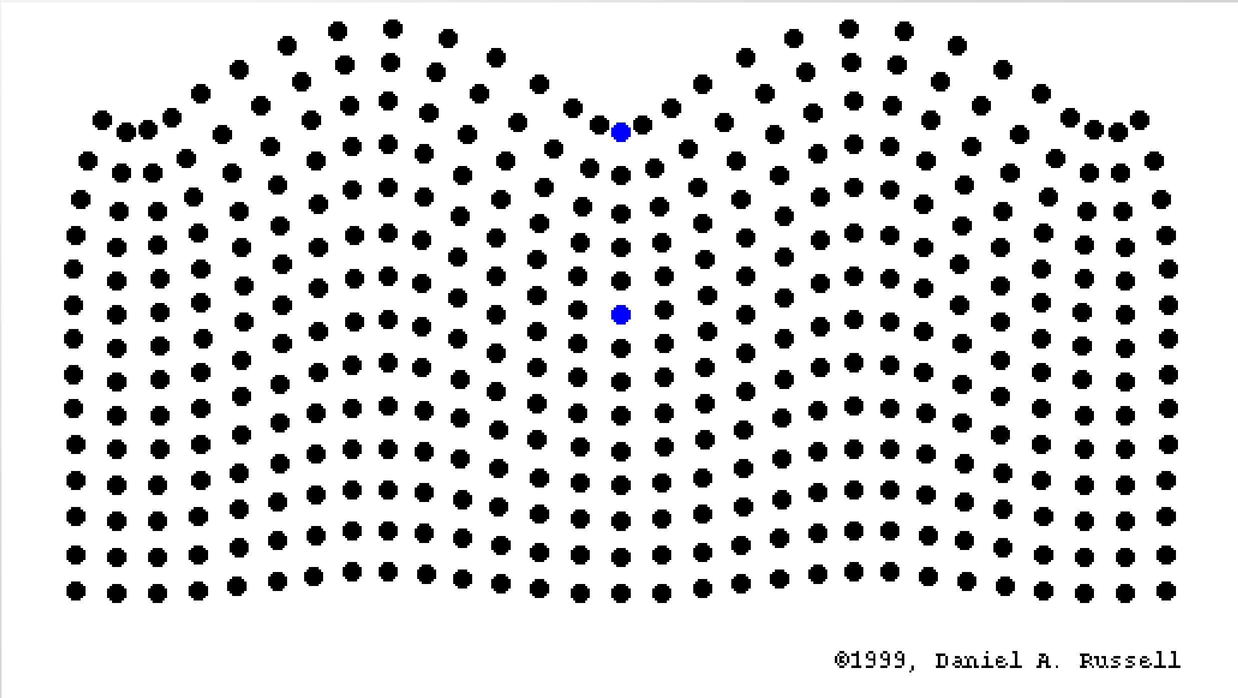
4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.2. Classification of surface acoustic waves

4.4.2.1. Rayleigh waves

E. Visualization of particle movements



4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.2. Classification of surface acoustic waves

4.4.2.1. Rayleigh waves

F. Danger for structures

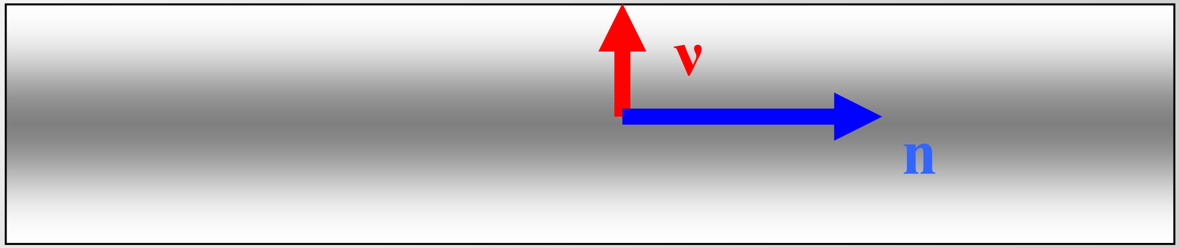
4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.2. Classification of surface acoustic waves

4.4.2.2. Lamb waves

A. Basic definition



Definition

Lamb waves propagate in a layer with either traction-free, clamped or mixed boundary conditions imposed on the outer surfaces of a layer.

Remark

These waves were discovered by Horace Lamb in 1917

4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.2. Classification of surface acoustic waves

4.4.2.2. Lamb waves

B. The main properties

- In contrast to Rayleigh waves, Lamb waves are **highly dispersive**, that means the the phase speed depends upon frequency or wavelength.
- There can be an infinite number of Lamb waves propagating with the same phase speed and differing by the frequency.
- Lamb waves can travel with both sub, intermediate, and supersonic speed.

Remark

After excitation, the most energy is transferred by the two lowest modes (symmetric and flexural).

4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.2. Classification of surface acoustic waves

4.4.2.2. Lamb waves

C. Possible applications

Due to their highly dispersive nature these waves are quite often used in NDT of possible defects in beams, plates, slabs, and rails.

Example

Evaluation of defects in rails

Rail head

Seismometer

Crack determination in rails

Accelerometer on Rail

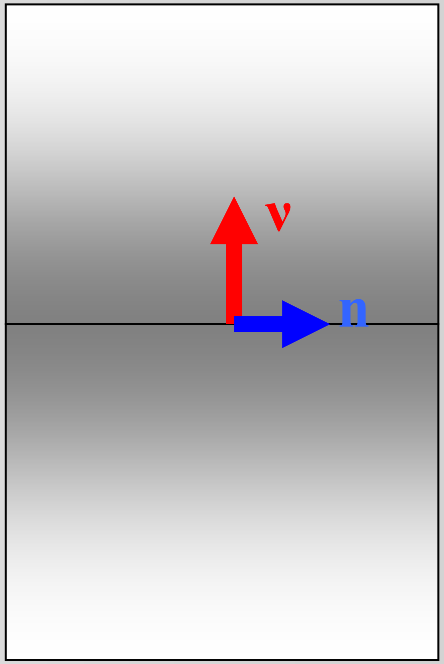
4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.2. Classification of surface acoustic waves

4.4.2.3. Stoneley waves

A. Basic definition



Definition

Stoneley waves are the waves traveling on an interface between two contacting half-spaces.

Remark

These waves were discovered and described by Robert Stoneley in 1924

4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.2. Classification of surface acoustic waves

4.4.2.3. Stoneley waves

B. The main properties

- As Rayleigh waves, Stoneley waves **are not dispersive** (their phase speed does not depend upon frequency or wavelength.
- For Stoneley waves a **uniqueness theorem** can be proved, stating that for arbitrary anisotropic and elastic halfspaces in a contact, there can be no more than one Stoneley wave.
- Generally, Stoneley waves propagate with the subsonic speed.

Remark

Not any contacting halfspaces may have Stoneley wave. Conditions for existence for two isotropic halfspaces in a contact were obtained by Stoneley.

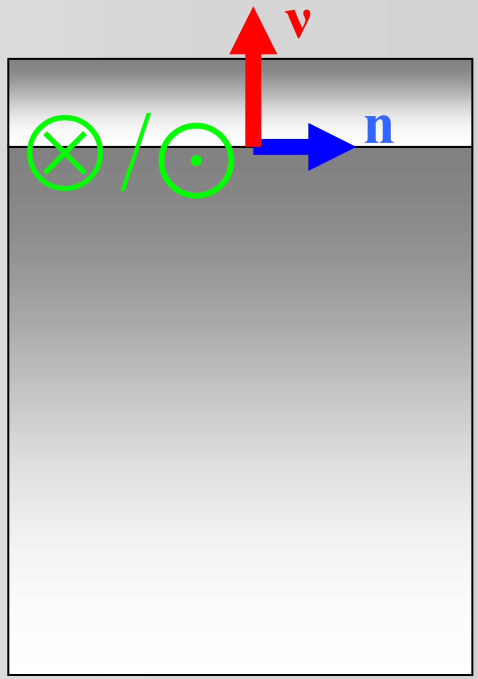
4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.2. Classification of surface acoustic waves

4.4.2.4. Love waves

A. Basic definition



Definition

Love waves are the waves traveling on an interface between two contacting half-spaces.

Remark

These waves were discovered and described by Augustus Love in 1911

4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.2. Classification of surface acoustic waves

4.4.2.4. Love waves

B. The main properties

- As Lamb waves, Love waves are **highly dispersive**, that means the their phase speed depends upon frequency or wavelength.
- There can be an infinite number of Love waves propagating with the same phase speed and differing by the frequency.
- Love waves can travel with the **subsonic** speeds for the **halfspace**.

Remarks

- It is assumed that Love wave attenuates with depth in the halfspace.

Accelerometer on Rail

- Not any layer and the contacting halfspace may possess Love waves.
- Conditions for existence for both *isotropic* layer and halfspace were obtained by Love.

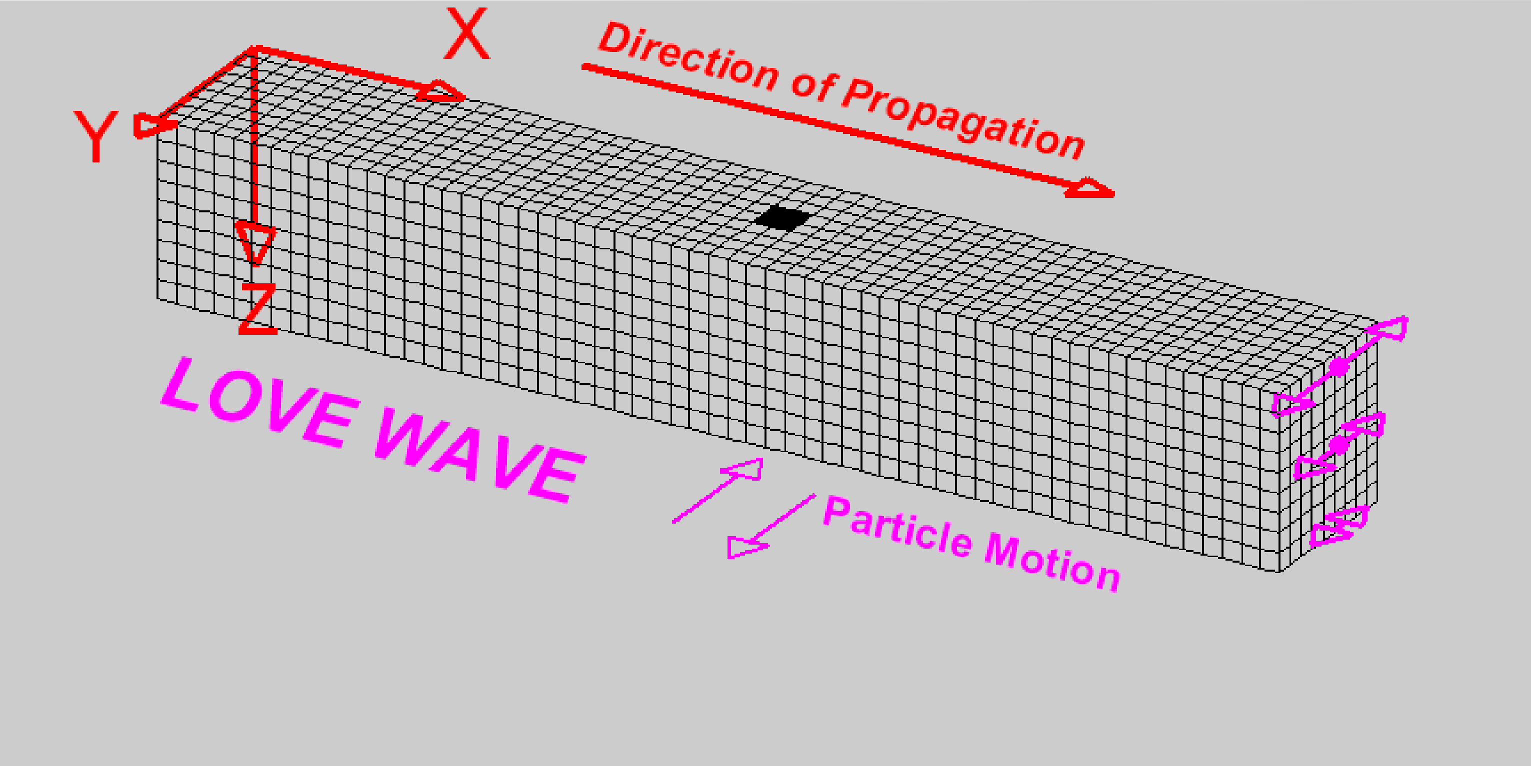
4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.2. Classification of surface acoustic waves

4.4.2.4. Love waves

C. Visualization of Love waves



Rail

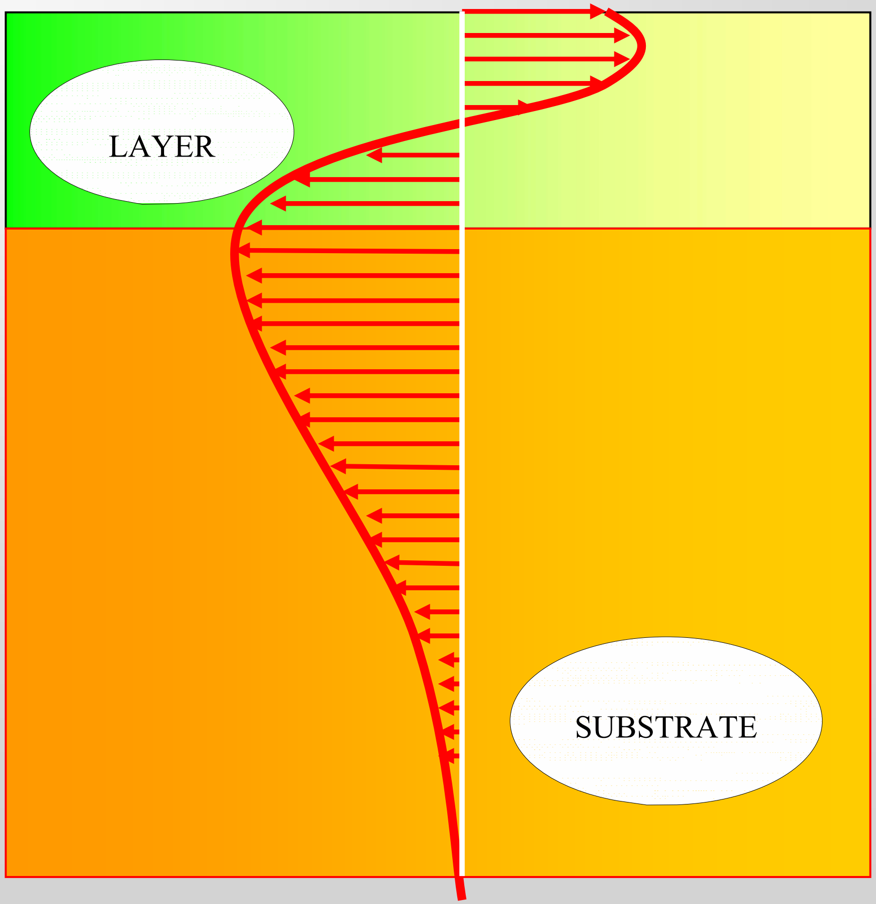
4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.2. Classification of surface acoustic waves

4.4.2.4. Love waves

D. Polarization of Love waves



4. Theoretical methods in acoustical studies

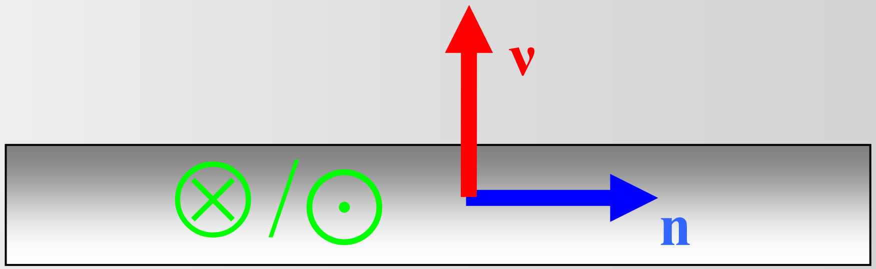
4.4. Surface acoustic waves

4.4.2. Classification of surface acoustic waves

4.4.2.5. SH waves

A. Basic definitions

No person is associated with



Definition

These waves travel in a layer or possibly several contacting layers, and have the transverse horizontal polarization.

Remark

Accelerometer on

As Lamb and Love waves, the SH waves are highly dispersive.

4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.2. Classification of surface acoustic waves

4.4.2.5. SH waves

B. The main properties

- As Lamb and Love waves, SH waves are **highly dispersive**, that means the their phase speed depends upon frequency or wavelength.
- There can be an infinite number of SH waves propagating with the same phase speed and differing by the frequency.
- SH waves can travel with both **supersonic** and **subsonic** speed (the subsonic speed cannot be achieved in a layer with the minimal shear bulk wave speed).

Accelerometer on
Rail

4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.2. Classification of surface acoustic waves

4.4.2.6. Speeds of bulk, Rayleigh, and Love waves

Generally (with some exceptions) the phase speeds satisfy the following conditions:

$$c_{longitudinal}^{bulk} > c_{transverse}^{bulk} > c_{Rayleigh} > c_{Love} \; *$$

Remark

* Strictly speaking, there is no single value for Love waves, as these waves are dispersive, and their speed satisfies the condition:

$$\left(c_{transverse}^{bulk}\right)_{layer} < c_{Love} < \left(c_{transverse}^{bulk}\right)_{substrate}$$

It is interesting to note, that there are following relations between the transmitted energy:

$$E_{Love} \sim E_{Rayleigh} >> E_{bulk}^{Transverse} \sim E_{bulk}^{Longitudinal}$$

4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.3. Mathematical methods for analyzing surface acoustic waves

4.4.3.1. Representations for the displacement field

$$\mathbf{u}(\mathbf{x},t) = \sum_{k=1}^{6} \mathbf{f}_k(x')e^{ir(\mathbf{n}\cdot\mathbf{x}-ct)}$$

Where

$x' = \nu \cdot \mathbf{x}$, thus, it is a coordinate along vector ν

$\mathbf{f}_k$ is the unknown function specifying variation of diplacements

$\mathbf{u}$ is a displacement field

$\mathbf{x}$ is a space variable

t is time

r is the wave number ($r = 2\pi / l$, or $r = \omega / c$)

$\mathbf{n}$ is direction of propagation ($\mathbf{n}$ is the unit vector)

c is the phase speed

4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.3. Mathematical methods for analyzing surface acoustic waves

4.4.3.2. Christoffel equations

Substituting representation for the surface wave into differential equations of motion, and performing necessary differentiation, yield the Christoffel equations for surface waves:

$$\left[\mathbf{A}(\nu)\partial_{x'}^{2} + 2\operatorname{sym}\left(\nu\cdot\mathbf{C}\cdot\mathbf{n}\right)\partial_{x'} + \mathbf{A}(\mathbf{n}) - \rho c^{2}\mathbf{I}\right]\cdot\mathbf{f}(x') = 0$$

Remark

- Thus constructed equation is the **matrix ODE** of the second order
- The unique method of constructing the solution is to reduce this equation to the matrix ODE of the first order.

4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.3. Mathematical methods for analyzing surface acoustic waves

4.4.3.3. Complex six-dimensional formalism

A. The main ODE in the complex six-dimensional form

Reducing to the ODE of the first order can be done by introducing a new (vector-valued) function:

$$\mathbf{w}(x') = \partial_{x'} \mathbf{f}(x')$$

Then, the Christoffel equation becomes

$$\partial_{x'} \begin{pmatrix} \mathbf{f} \\ \mathbf{w} \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{A}^{-1}(\nu) \cdot \left(\mathbf{A}(\mathbf{n}) - \rho c^2 \mathbf{I} \right) & -2\mathbf{A}^{-1}(\nu) \cdot \operatorname{sym}\left(\nu \cdot \mathbf{C} \cdot \mathbf{n} \right) \end{pmatrix} \cdot \begin{pmatrix} \mathbf{f} \\ \mathbf{w} \end{pmatrix}$$

Remark

The last first-order ODE is known as the six-dimensional complex formalism.

4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.3. Mathematical methods for analyzing surface acoustic waves

4.4.3.3. Complex six-dimensional formalism

B. The general solution

The last first-order gives 6 linearly independent six dimensional vector-functions, allowing us to construct the general solution:

$$\mathbf{g}_{6-\mathrm{dim}}(x') = \sum_{k=1}^{6} C_k \begin{pmatrix} \mathbf{f}_k(x') \\ \mathbf{w}_k(x') \end{pmatrix}$$

Remark

The unknown coefficients C_k are defined by substituting this solution into boundary, interface, and Sommerfield's attenuation conditions

4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.3. Mathematical methods for analyzing surface acoustic waves

4.4.3.3. Complex six-dimensional formalism

C. Structure of the solution

$$\mathbf{f}_k(x') = \mathbf{m}_k e^{ir\gamma_k x'}$$

Where

$\mathbf{m}_k$ is the amplitude of the partial wave

γ_k is the Christoffel parameter of the partial wave

Remark

The exponential term $e^{ir\gamma_k x'}$

ensures either exponential growth (if $\mathrm{Im}(\gamma_k)<0$),

or exponential decay (if $\mathrm{Im}(\gamma_k)>0$),

or a periodic variation (if $\mathrm{Im}(\gamma_k)=0$).

4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.3. Mathematical methods for analyzing surface acoustic waves

4.4.3.3. Complex six-dimensional formalism

D. History of constructing this formalism

Eshelby	~ 1956	?
Stroh	1962	Development of the sextic formalism
Barnett & Lothe	1973-76	Analysis of Rayleigh waves by the sextic formalism
Chadwick & Smith	1977	Foundations of the sextic formalism
Alshits	1977	Applications to leakage waves
Chadwick & Ting	1987	Structure of the Barnett-Lothe tensors
Mase	1987	Rayleigh wave speed in transversely isotropic media
Ting & Barnett	1997	Classification of surface waves in crystals

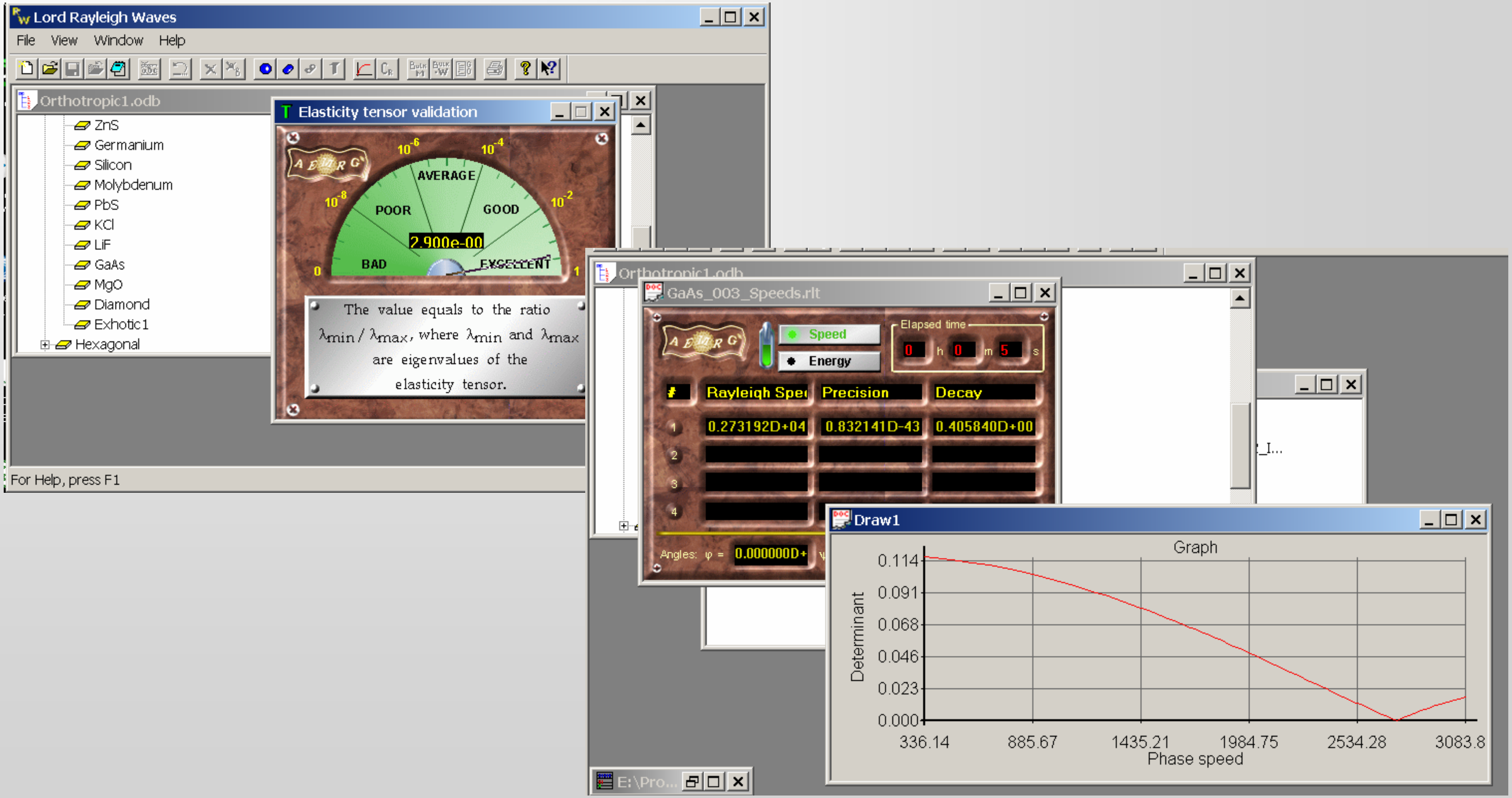
4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.3. Mathematical methods for analyzing surface acoustic waves

4.4.3.4. An example of finding the solutions

A. “Lord Rayleigh” Rayleigh wave simulation software



4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.3. Mathematical methods for analyzing surface acoustic waves

4.4.3.4. An example of finding the solutions

B. Speed of Rayleigh waves for some materials

Material	Syngony, direction	Rayleigh wave speed, m/sec
GaAs, Gallium Arsenide	Cubic, [001]	2731.8
Ge, Germanium	Cubic, [100]	2929.9
InSb, Indium Antimonide	Cubic, [001]	1833.3

4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.4. Problem of "forbidden" directions for Rayleigh waves

For a long time the main problem related to Rayleigh wave propagating was finding conditions at which such a wave cannot propagate (problem of "forbidden" directions).

Do such "forbidden" directions exist?

Theorem of existence for Rayleigh waves:

Barnett and Lothe, 1973-76

Chadwick, 1975-85

Ting, 1983-96

But, in 1998-2002 a type of Non-Rayleigh waves was theoretically observed and constructed explicitly.

4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.5. Anomalous solutions for Rayleigh waves (Non-Rayleigh wave type waves)

4.4.5.1. Conditions for appearing the anomalous waves

These waves correspond to appearing the **Jordan blocks** in a six-dimensional matrix associated with the Christoffel equation.

Structure of a Non-Rayleigh type wave

$$\mathbf{u}(\mathbf{x},t) = \left(\mathbf{m}_1 + \mathbf{m}_2^* x'\right) e^{ir\gamma x'} e^{ir(\mathbf{n}\cdot\mathbf{x} - ct)}$$

Where

$\mathbf{m}_2^*$ is the generalized eigenvector

4. Theoretical methods in acoustical studies

4.4. Surface acoustic waves

4.4.5. Anomalous solutions for Rayleigh waves (Non-Rayleigh wave type waves)

4.4.5.2. Jordan blocks

Marie Ennemond Camille

Jordan

1838 - 1922

$$\mathbf{J} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

4. Theoretical methods in acoustical studies

4.5. Surface acoustic waves in multilayered media

4.5.1. Global matrix method

The main idea

Constructing a “global” matrix combining all the equations for the particular layers:

$$M \equiv \begin{pmatrix} A_1^+ & & & & \\ A_1^- & A_2^+ & & & \\ & A_2^- & A_3^+ & & \\ & & A_3^- & A_4^+ & \\ & & & A_4^- & A_5 \end{pmatrix} \quad \Rightarrow \quad \det(M) = 0$$

Originator

Suggested by Leon Knopoff (1964)

4. Theoretical methods in acoustical studies

4.5. Surface acoustic waves in multilayered media

4.5.2. Transfer matrix method

The main idea

Constructing the "transfer" matrix allowing us to express boundary conditions at the bottom boundary in terms of the coefficients of the uppermost layer

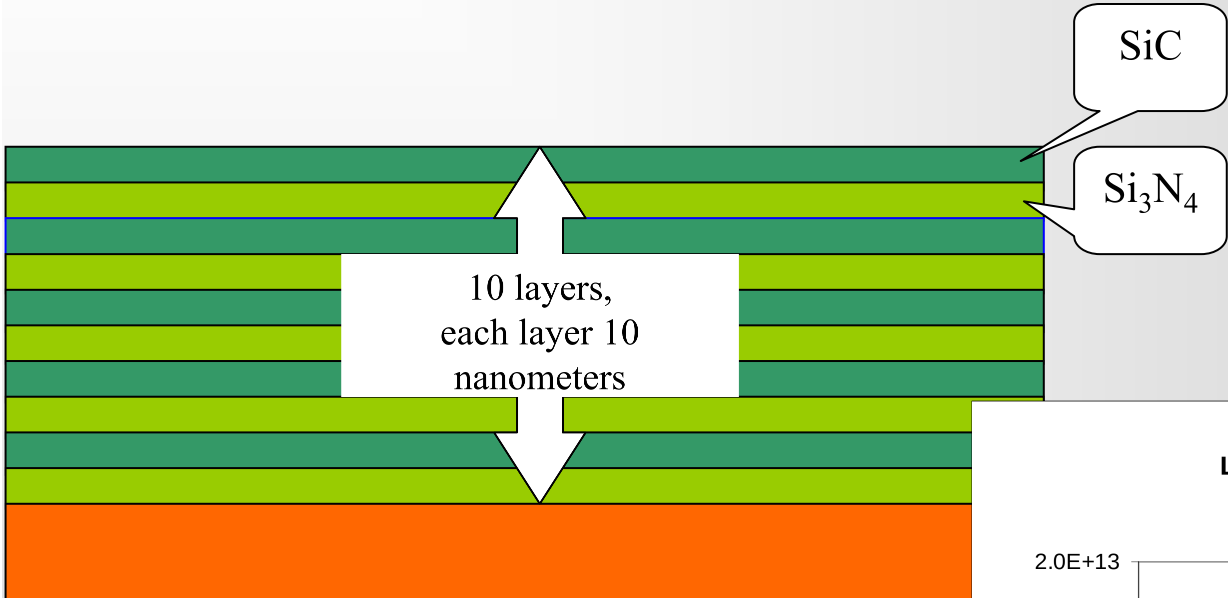
$$M \equiv A_1^+ \cdot \left(A_1^-\right)^{-1} \cdot A_2^+ \cdot \left(A_2^-\right)^{-1} \cdot \ldots \cdot A_n^+ \qquad \Rightarrow \qquad \det(M) = 0$$

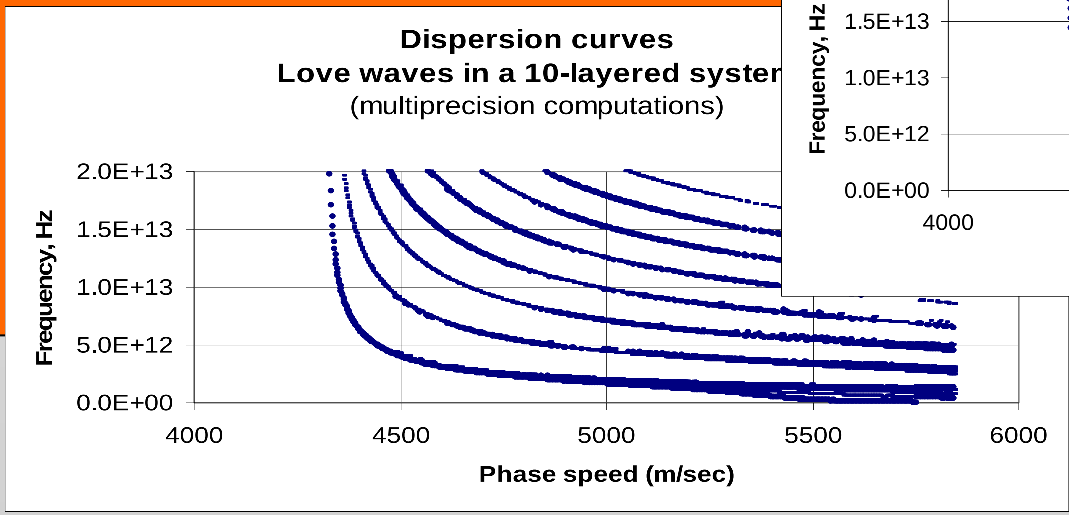
Originator

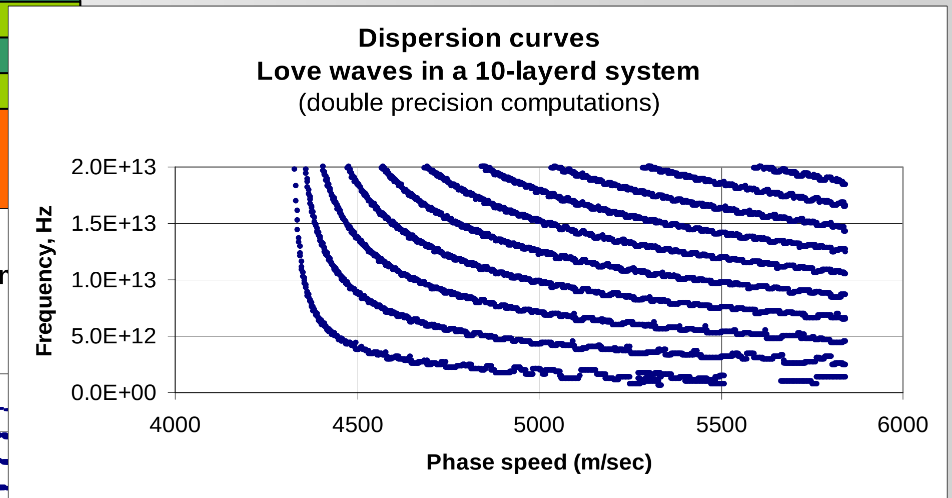
Suggested by Thomson (1950) and Haskell (1953)

4. Theoretical methods in acoustical studies

4.5. Surface acoustic waves in multilayered media
4.5.3. Role of multiprecision computations





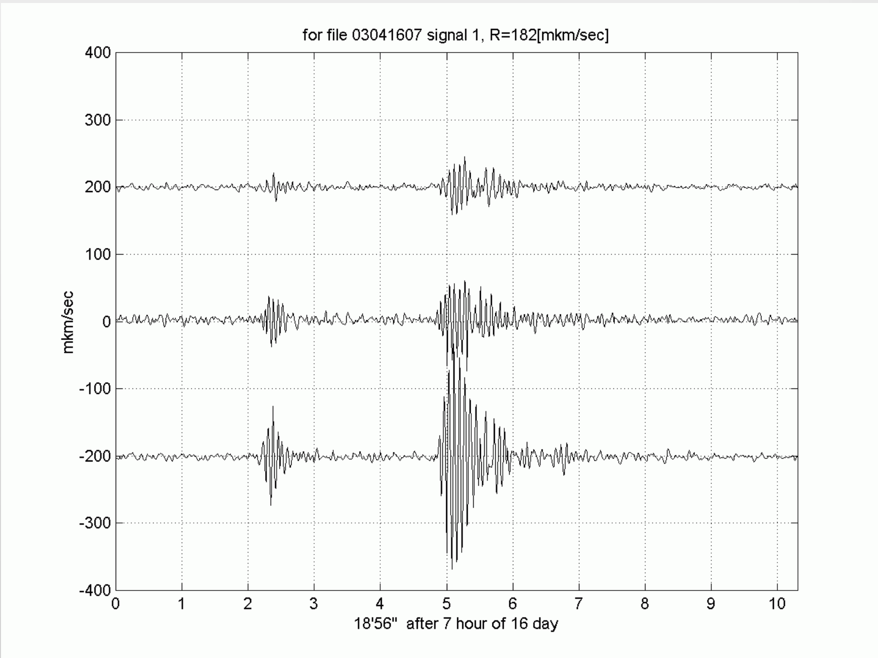


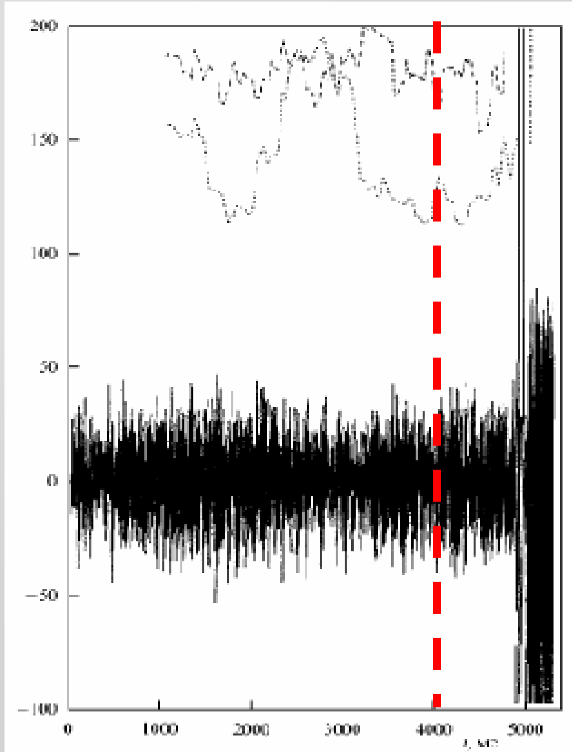
Part III. Engineering applications

5. Deterministic mathematical methods used for predicting earthquakes

5.1. The main problem of predicting analysis

Analysis of a seismogram $\Rightarrow$ conclusion on possibility of the event





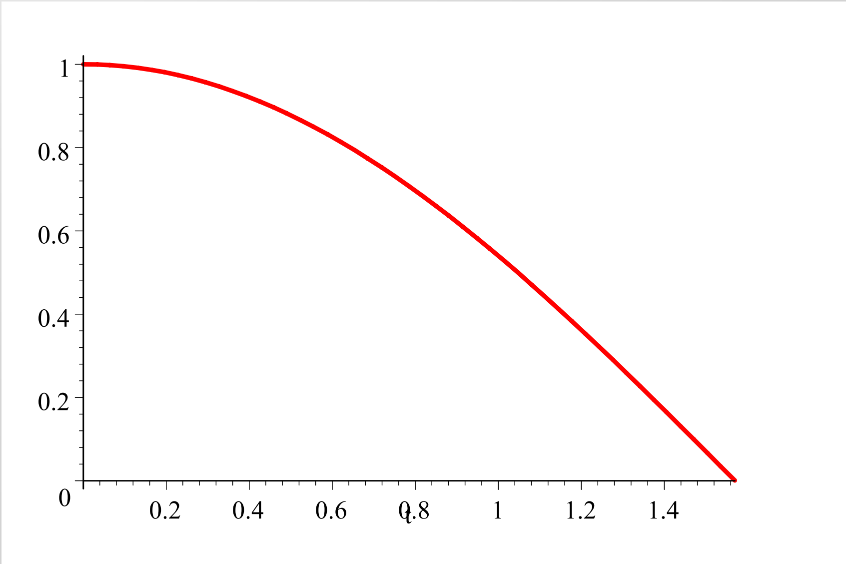
5. Deterministic mathematical methods used for predicting earthquakes

5.2. Fourier and wavelet analyses

5.2.1. Fourier transforms

5.2.1.1. Example of a function to be extrapolated

$$\cos(t) \text{ at } \; t \in \left[0,\ \pi/2\right]$$



5. Deterministic mathematical methods used for predicting earthquakes

5.2. Fourier and wavelet analyses

5.2.1. Fourier transforms

5.2.1.2. Fourier analysis and synthesis

$$f(t) \sim \sum_{k=-\infty}^{\infty} c_k \exp(2\pi i\, kt / p)$$

Where

$$c_k = \frac{1}{p} \int_{t_0}^{t_0+p} f(\tau) \exp(2\pi i\, k\tau / p) \; d\tau$$

Fourier series for the given function

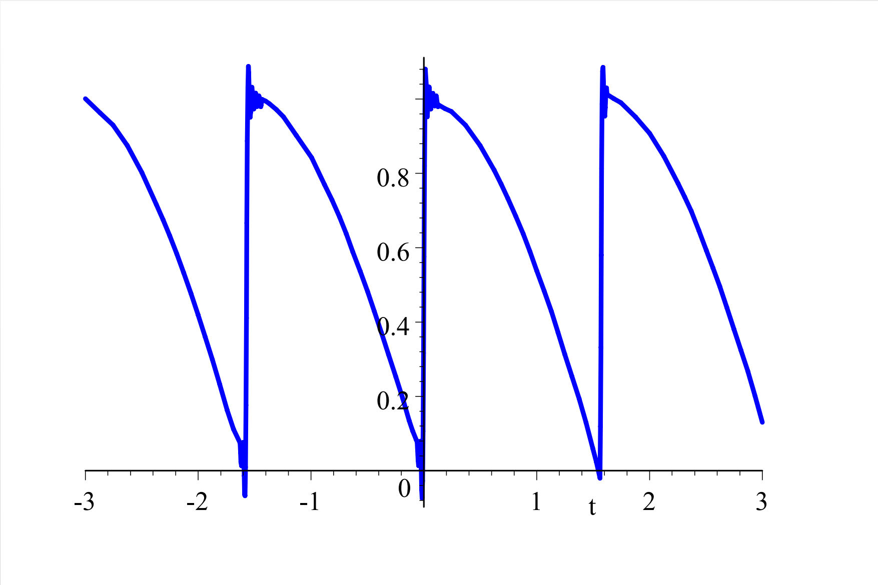
$$f(t) \sim \sum_{k=-\infty}^{\infty} \frac{2(4ik - \exp(2ik\pi))}{\pi(16k^2 - 1)} \exp(4i\, kt)$$

5. Deterministic mathematical methods used for predicting earthquakes

5.2. Fourier and wavelet analyses

5.2.1. Fourier transforms

5.2.1.3. Resulting function



Summation of Fourier series for following initial function

$$\cos(t) \text{ at } t \in [0, \pi/2]$$

5. Deterministic mathematical methods used for predicting earthquakes

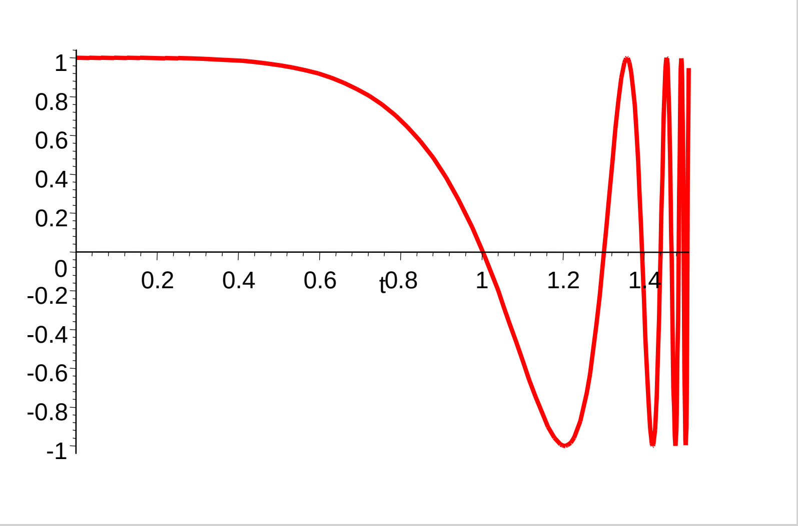
5.2. Fourier and wavelet analyses

5.2.1. Fourier transforms

5.2.1.4. Shortcomings

Fourier analysis (series) is inapplicable to predicting:

- non-periodic processes
- processes with the unknown period(s)
- processes with variable periods



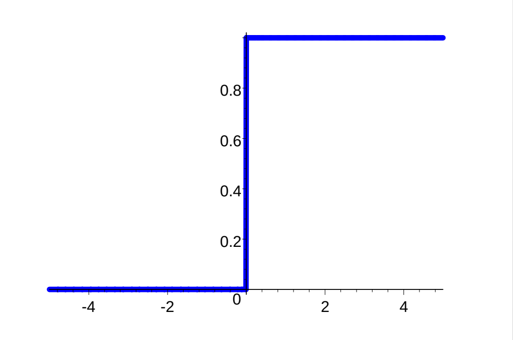
5. Deterministic mathematical methods used for predicting earthquakes

5.2. Fourier and wavelet analyses

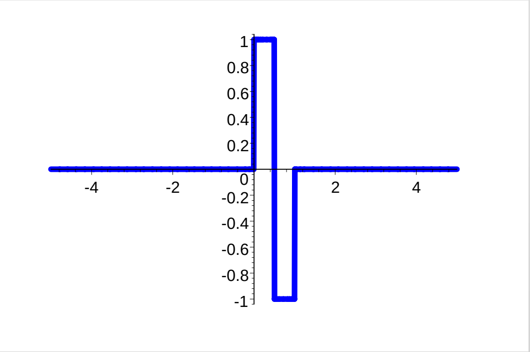
5.2.2. Wavelet transforms

5.2.2.1. The main ideas

Heaviside step function



Haar's generic wavelet



Wavelets

$$\psi_{j,k}(t) = 2^{j/2}\psi_H(2^j t - k)$$

$$\psi_H(t) = H(t)H(1-2t) - H(2t-1)H(1-t)$$

5. Deterministic mathematical methods used for predicting earthquakes

5.2. Fourier and wavelet analyses

5.2.2. Wavelet transforms

5.2.2.2. Basic properties

Orthogonality:

$$\int \psi_{j,k}(t)\psi_{m,n}(t)\,dt = 0, \quad \text{at} \quad j \neq m \quad \underline{\underline{\text{or}}} \quad k \neq n$$

Normality:

$$\int \left(\psi_{j,k}(t)\right)^2 dt = 1$$

5. Deterministic mathematical methods used for predicting earthquakes

5.2. Fourier and wavelet analyses

5.2.2. Wavelet transforms

5.2.2.3. Shortcomings and advantage

Shortcomings:

Wavelet analysis is inapplicable to predicting:

- non-periodic processes
- processes with the unknown period(s)

Advantage:

Wavelet analysis is well suited for processes with the variable periods

5. Deterministic mathematical methods used for predicting earthquakes

5.3. Lagrange and Newton interpolating polynomials

5.3.1. Basic idea (assumption of analyticity)

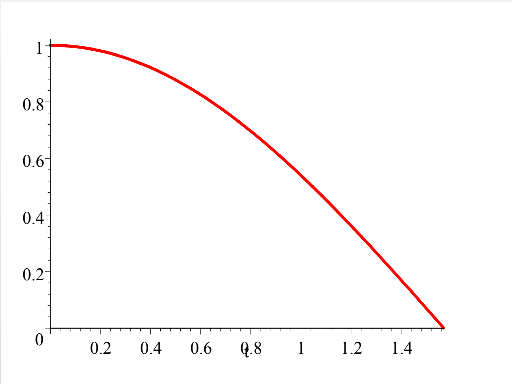
Assume that the function to be extrapolated is analytic in a vicinity of the endpoint, then such a function can be expanded into convergent Taylor's series:

$$f(t) = \sum_{k=0}^{\infty} \frac{f^{(k)}(t_b)}{k!} (t - t_b)^k$$

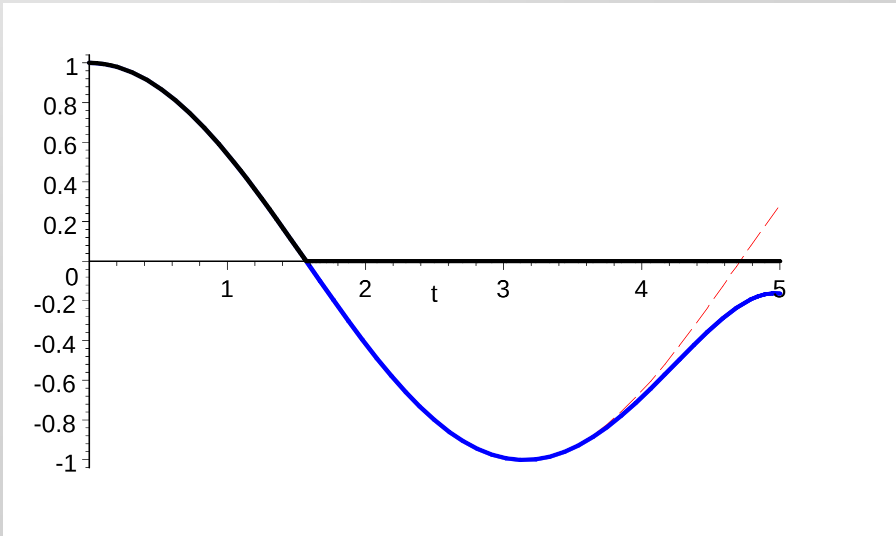
5. Deterministic mathematical methods used for predicting earthquakes

5.3. Lagrange and Newton interpolating polynomials

5.3.2. An example of cosine function



Our cosine function at the endpoint $\pi/2$ can be expanded into Taylor's series, which after truncating to the first 10 terms gives the following:



5. Deterministic mathematical methods used for predicting earthquakes

5.3. Lagrange and Newton interpolating polynomials

5.3.3. Transition to interpolating polynomials

Unfortunately, in most of practical situations we do not know analytical expressions for the function to be interpolated

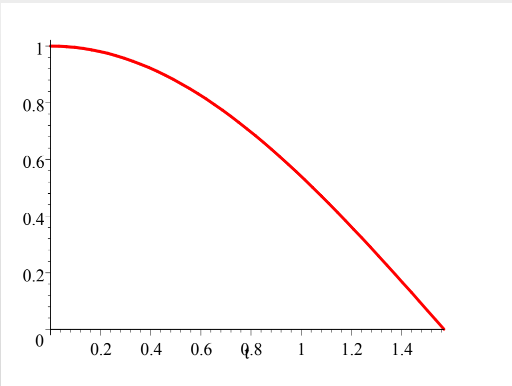
$$\Downarrow$$

But, we can try to construct an interpolating polynomial, and then find extrapolation by the interpolating polynomial

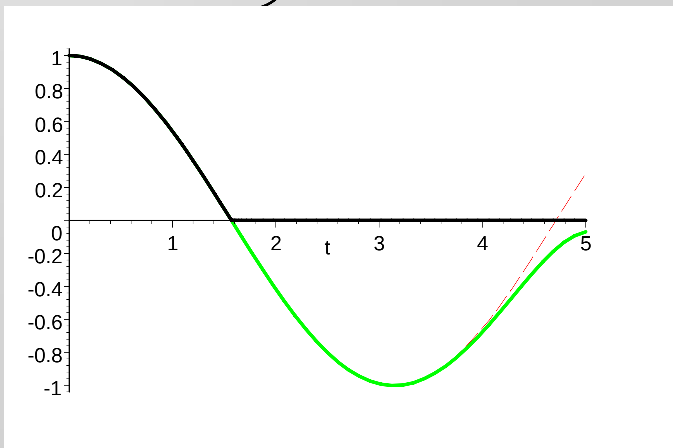
5. Deterministic mathematical methods used for predicting earthquakes

5.3. Lagrange and Newton interpolating polynomials

5.3.4. Interpolating polynomial for cosine function



Lagrange interpolating polynomial of the 10-th order



5. Deterministic mathematical methods used for predicting earthquakes

5.4. Role of multiprecision calculations in the predicting analyses

5.4.1. An example of a two-term polynomial

$$P(x) = x^{10} - 10x^9$$

Roots:

$$x^9(x-10) = 0 \quad \Rightarrow \quad x_1 = 0, \quad x_2 = 10$$

5. Deterministic mathematical methods used for predicting earthquakes

5.4. Role of multiprecision calculations in the predicting analyses

5.4.2. A small perturbation

$$P_{\varepsilon}(x) = x^{10} - 10.00001\,x^{9}$$

$$\Downarrow$$

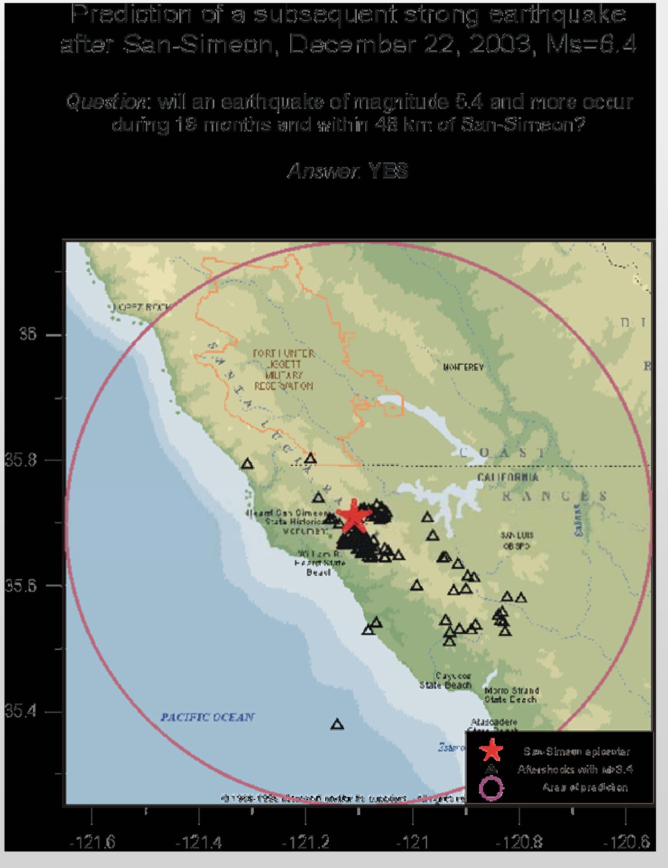
$$P_{\varepsilon}(10) = -10000$$

while

$$P(10) = 0$$

5. Deterministic mathematical methods used for predicting earthquakes

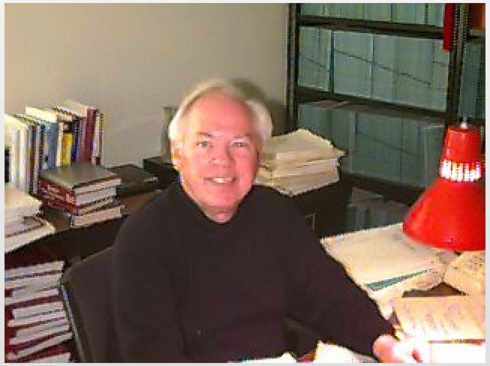
5.5. Example of predicting earthquakes in US



Prediction dated Dec 23 2003 is based on the analysis of small vibrations in the San-Simeon region (CA)

6. Principles of creating seismic and vibration barriers

6.1. Rough surface, as a barrier for Rayleigh waves



Alexei A.Maradudin

Works on rough surfaces for Rayleigh waves 1976-78

The main results:

If a free surface is not flat, but contains some small periodic perturbations (of Lyapunov class), then the corresponding Rayleigh wave begins attenuate

The rate of attenuation depends upon frequency of Rayleigh wave

6. Principles of creating seismic and vibration barriers

6.2. Modifying surface layers for creating barriers against Love waves

6.2.1. The main principle (Actually, A.E.H.Love, 1911):

Love wave cannot propagate in an isotropic elastic layer perfectly connected to the isotropic elastic halfspace, if speed of propagation of bulk shear wave in the layer is greater than in the substrate:

$$\left(c_{transverse}^{bulk}\right)^{layer} > \left(c_{transverse}^{bulk}\right)^{substrate}$$

6. Principles of creating seismic and vibration barriers

6.2. Modifying surface layers for creating barriers against Love waves

6.2.2. Consequence

$$c_{transverse}^{bulk} = \sqrt{\frac{\mu}{\rho}}$$

&

Love's principle

$$\Rightarrow \quad \frac{\mu^{layer}}{\mu^{substrate}} > \frac{\rho^{layer}}{\rho^{substrate}}$$

$$\Downarrow$$

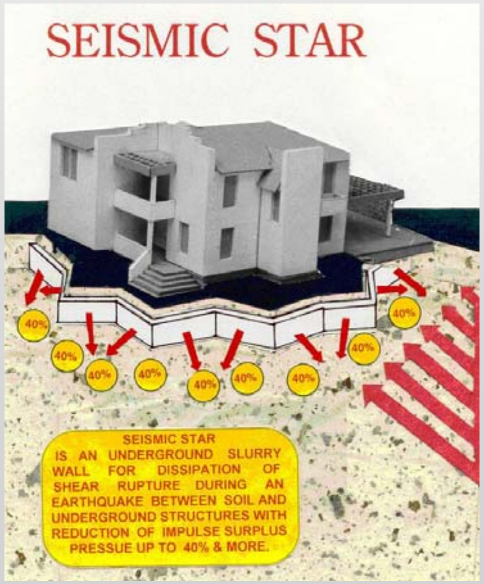
A condition for a Love wave barrier

6. Principles of creating seismic and vibration barriers

6.3. "Walls" in rocks and soils to prevent surface acoustic waves to propagate

6.3.1. Principle of reflection

6.3.1.1. An example of the reflecting barrier



Kalmatron Corp. with its star-shaped protection system

One of the obvious deficiencies:

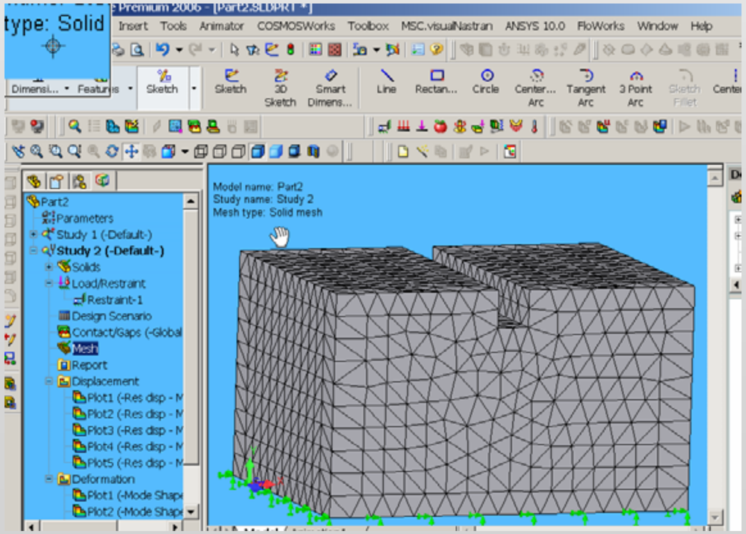
For a relatively large wavelength of seismic waves (10-6000m) the protected system should be at least the same depth

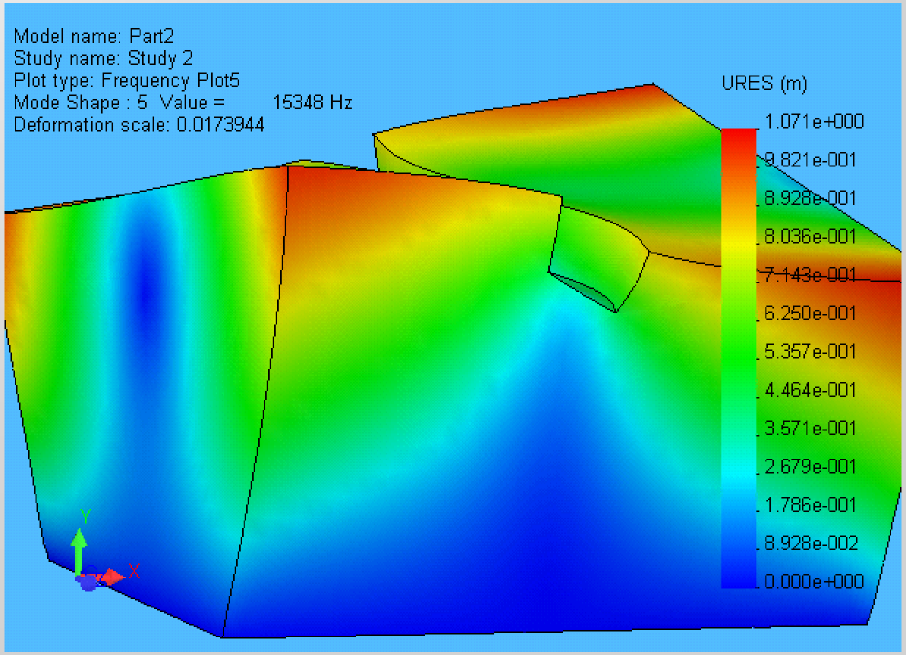
6. Principles of creating seismic and vibration barriers

6.3. "Walls" in rocks and soils to prevent surface acoustic waves to propagate

6.3.1. Principle of reflection

6.3.1.2. Some problems in creating reflecting seismic barriers



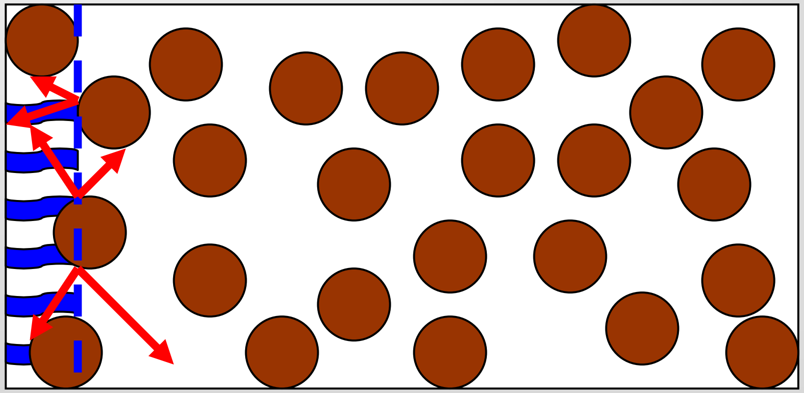


6. Principles of creating seismic and vibration barriers

6.3. "Walls" in rocks and soils to prevent surface acoustic waves to propagate

6.3.2. Principle of scattering

6.3.2.1. The main idea



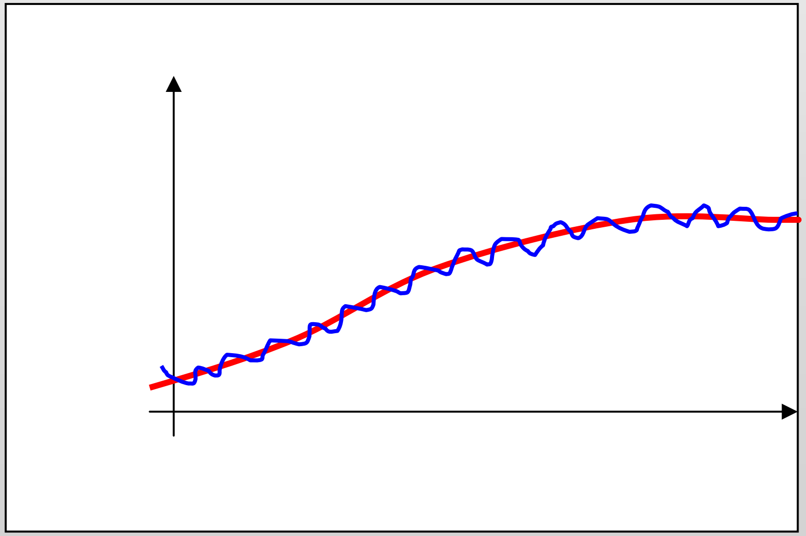
The best results with respect to scattering can be achieved, if inclusions are the closed pores.

6. Principles of creating seismic and vibration barriers

6.3. “Walls” in rocks and soils to prevent surface acoustic waves to propagate

6.3.2. Principle of scattering

6.3.2.2. Mathematical method



Two-scale asymptotic analysis

x – "slow" variable

X – "fast" variable

Relation between variables

$$X = \frac{1}{\varepsilon} x, \qquad \varepsilon \to 0$$

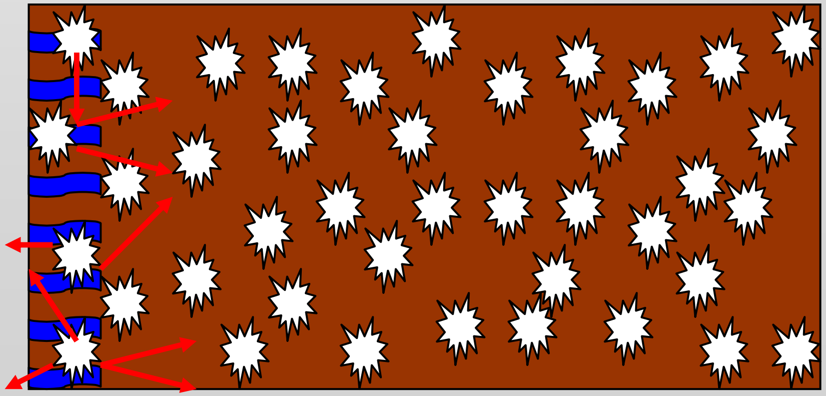
6. Principles of creating seismic and vibration barriers

6.3. "Walls" in rocks and soils to prevent surface acoustic waves to propagate

6.3.2. Principle of scattering

6.3.2.3. The main result

The best results with respect to the scattering effect are achieved, when the inclusions are pores

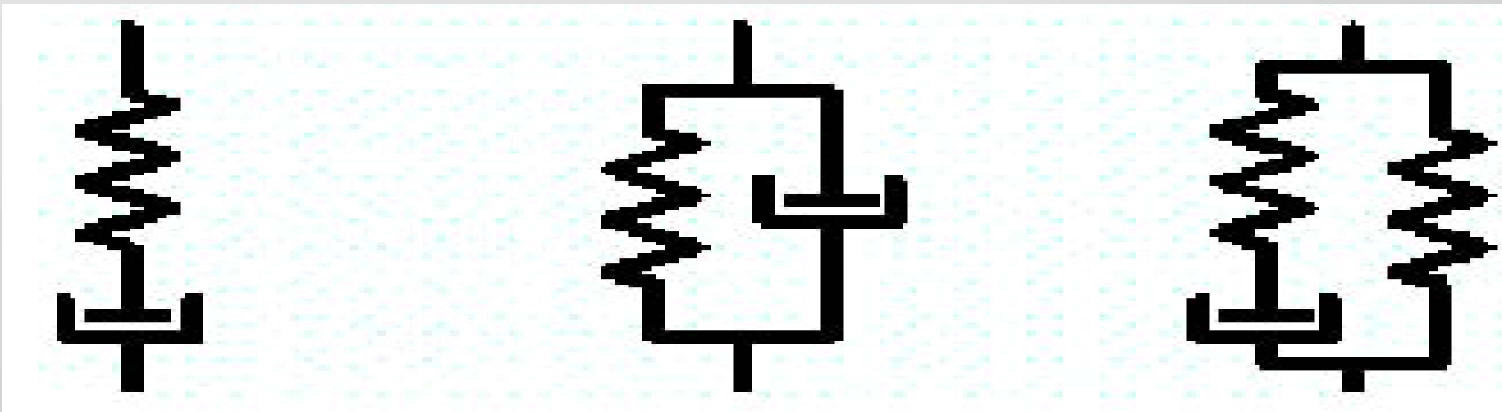


7. Viscoelastic dampers for vibration and seismic protection

7.1. Maxwell, Kelvin (Voigt), and standard elements

7.1.1. The main elements

Maxwell element Kelvin (Voigt) element Standard linear



7. Viscoelastic dampers for vibration and seismic protection

7.1. Maxwell and Kelvin (Voigt) elements

7.1.2. Differential equation for Kelvin's element

$$m\ddot{x} + c\dot{x} + kx = 0$$

m is mass

c is viscosity of a dashpot

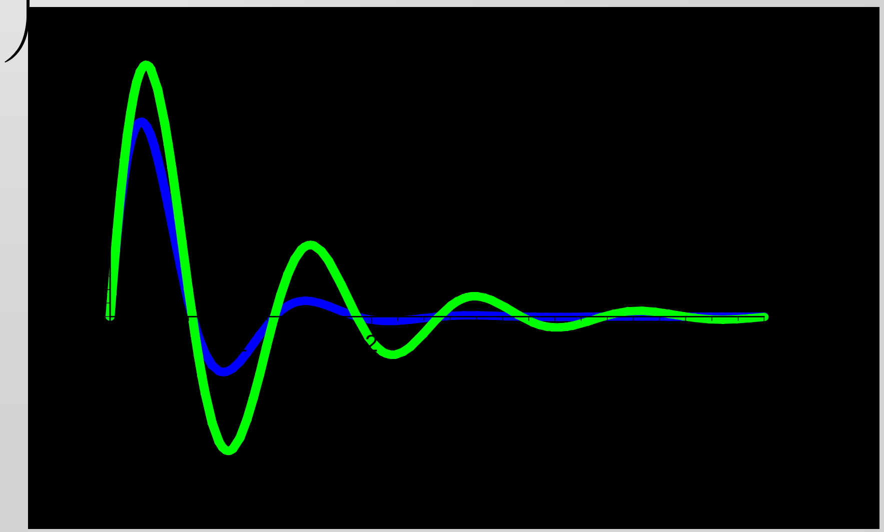
k is the spring rate

7. Viscoelastic dampers for vibration and seismic protection

7.1. Maxwell and Kelvin (Voigt) elements

7.1.3. The general solution of the equation for Kelvin's element (free vibrations)

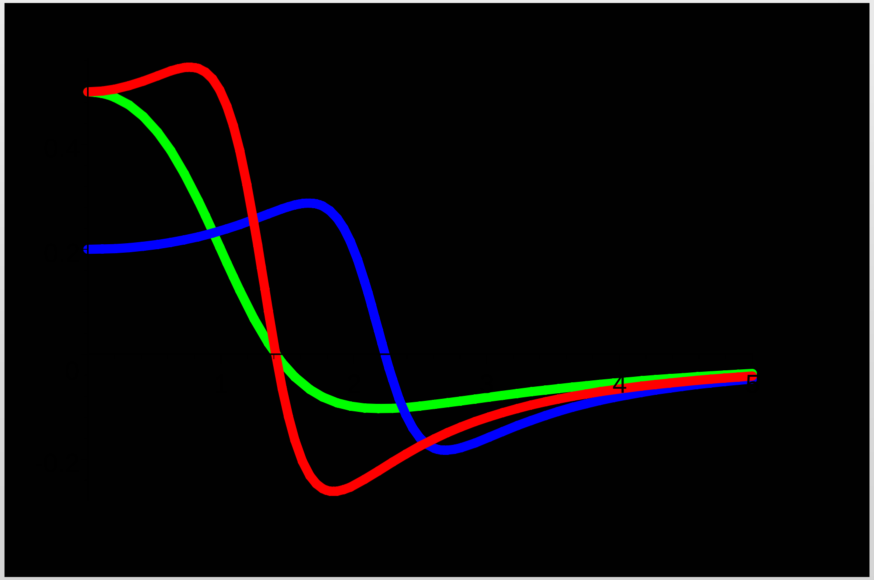
$$x = \exp\left(-\frac{c}{2m}t\right)\exp\left(i\sqrt{\frac{k}{m}-\left(\frac{c}{2m}\right)^2}\;t\right)$$



7. Viscoelastic dampers for vibration and seismic protection

7.1. Maxwell and Kelvin (Voigt) elements

7.1.4. Response to the oscillating loadings

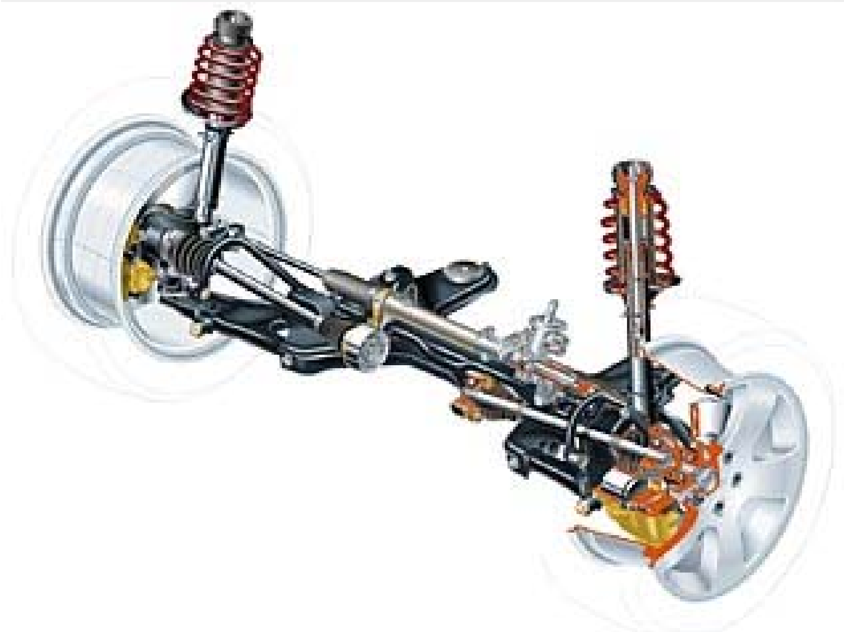


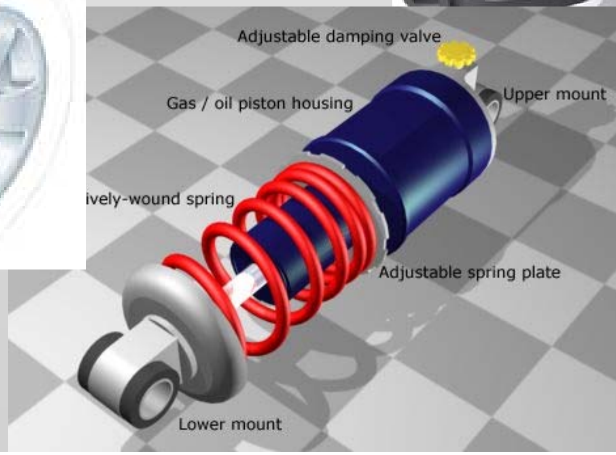
Dependence of the amplitude of oscillations upon frequency of the applied loading for the fixed damping system (Kelvin’s element)

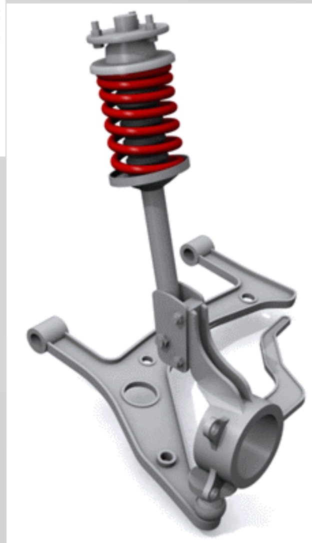
7. Viscoelastic dampers for vibration and seismic protection

7.2. Damping in automotive industry

7.2.1. Design of McPherson struts and suspensions





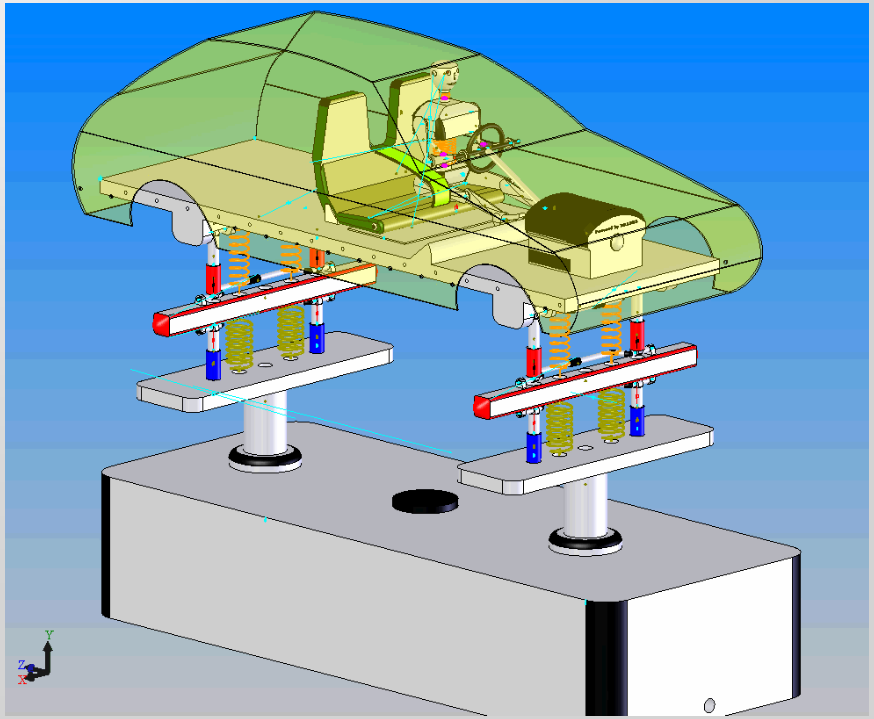


Derived from the “Bible of Car Suspensions”

7. Viscoelastic dampers for vibration and seismic protection

7.2. Damping in automotive industry

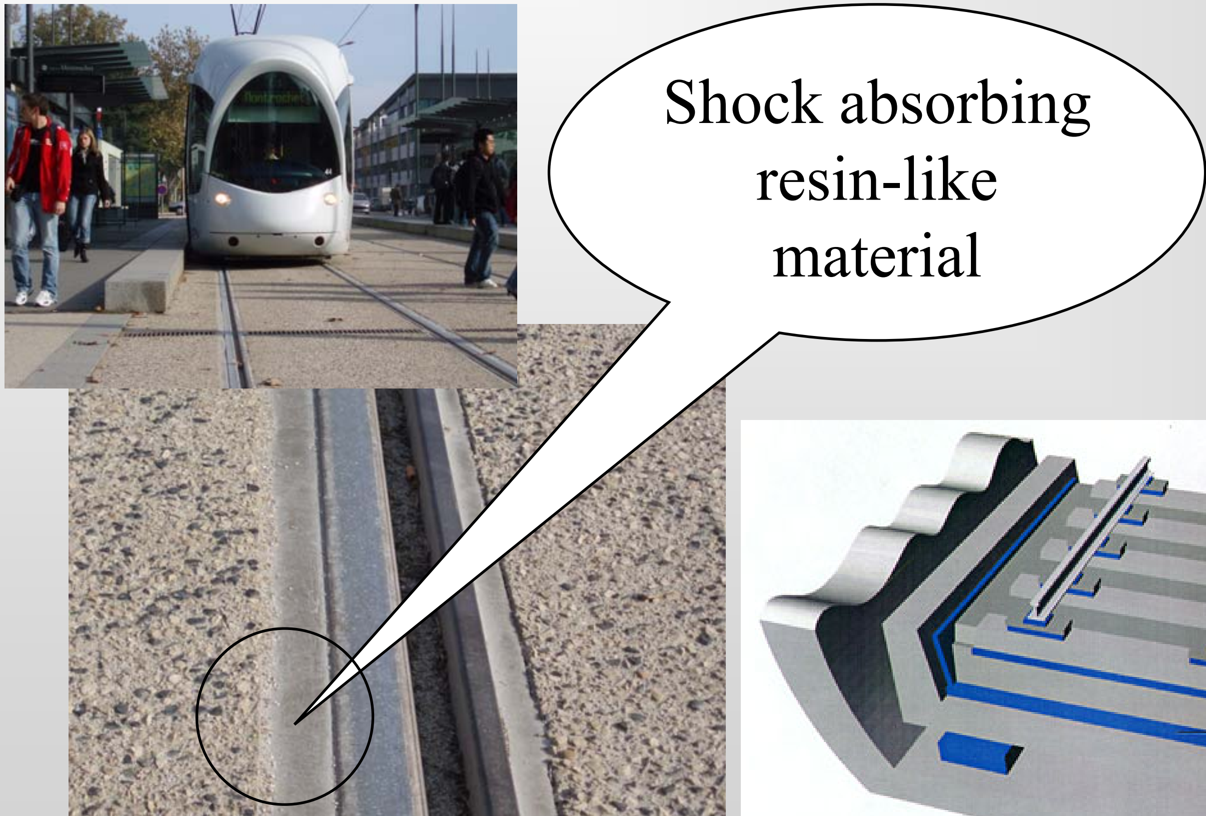
7.2.2. Example of unsuccessful suspension tuning

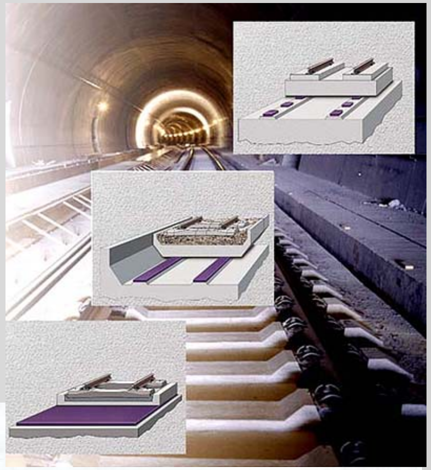


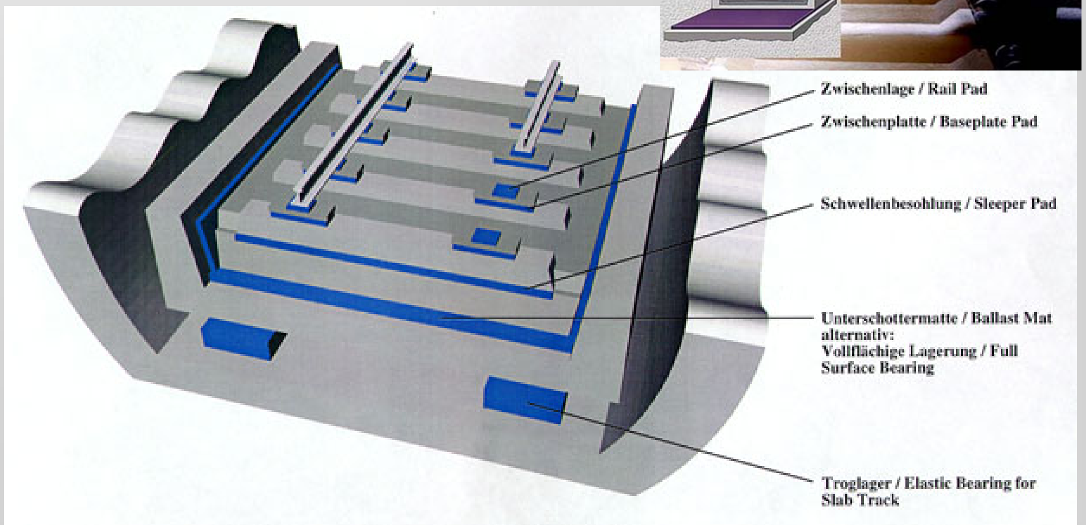
Derived from A. Kuznetsov diploma work
at MAMI Moscow Technical University

7. Viscoelastic dampers for vibration and seismic protection

7.3. Shock and vibration absorbers in railway engineering







7. Viscoelastic dampers for vibration and seismic protection

7.4. Vibration absorbers in bridge engineering

Resin-polymer vibration absorbers between span beams and columns (Bridge over the Rhone river, Villeurbanne, France)

7. Viscoelastic dampers for vibration and seismic protection

7.5. Shock and vibration absorbers in civil and industrial engineering

7.5.1. Dashpots (dampers) for seismic protection

7.5.1.1. Dashpots by “Taylor Devices” and “Scott Forge”

A. Dashpot design

Dashpots by “Taylor Devices” (NY, USA)

Dashpots by “Scot Forge” (IL, USA)

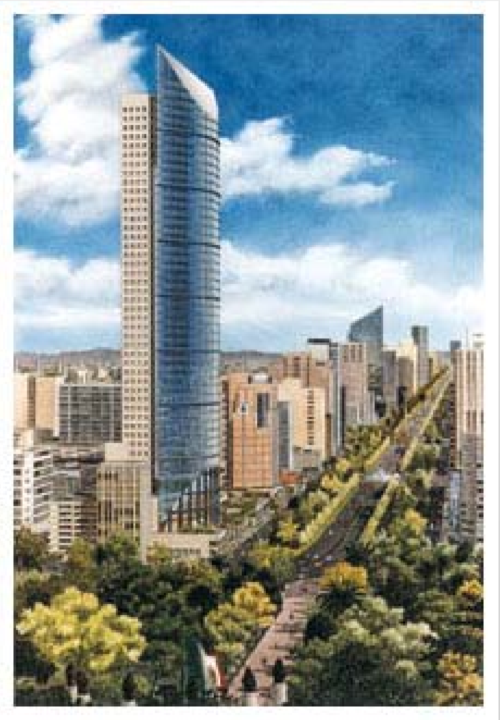
7. Viscoelastic dampers for vibration and seismic protection

7.5. Shock and vibration absorbers in civil and industrial engineering

7.5.1. Dashpots (dampers) for seismic protection

7.5.1.1. Dashpots by “Taylor Devices” and “Scott Forge”

B. “Torre Mayor” building equipped with dashpots



The 55-story Torre Mayor (Mexico city), meaning "Big Tower," is the tallest building in Latin America

On January 21, 2003, Mexico city experienced a 7.6 magnitude earthquake, but occupants of the building did not suffer from this earthquake.

7. Viscoelastic dampers for vibration and seismic protection

7.5. Shock and vibration absorbers in civil and industrial engineering

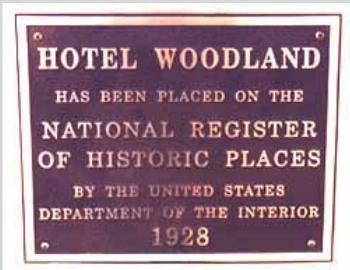
7.5.1. Dashpots (dampers) for seismic protection

7.5.1.1. Dashpots by “Taylor Devices” and “Scott Forge”

C. Other structure equipped with dashpots

Hotel Woodland in Woodland California, USA

Notice, that dashpots are installed in the upper part of the frame!



7. Viscoelastic dampers for vibration and seismic protection

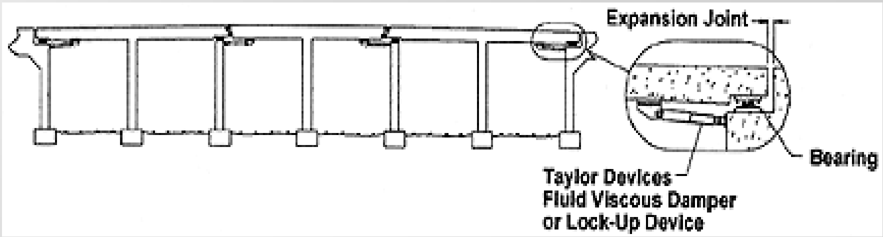
7.5. Shock and vibration absorbers in civil and industrial engineering

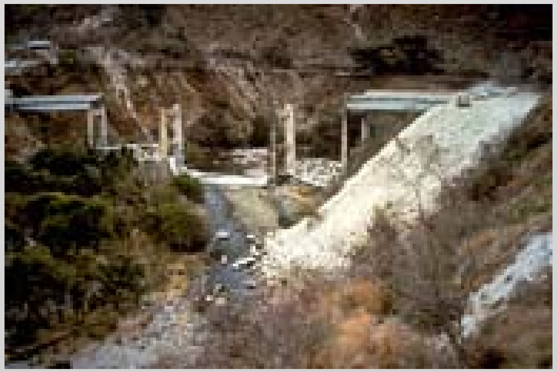
7.5.1. Dashpots (dampers) for seismic protection

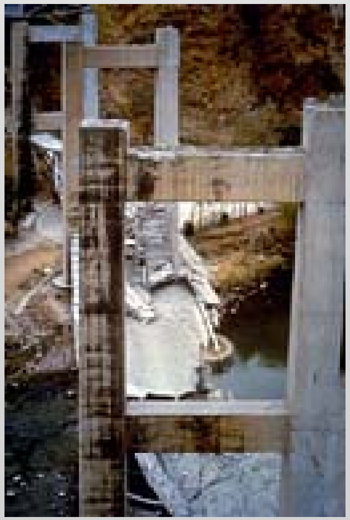
7.5.1.1. Dashpots by “Taylor Devices” and “Scott Forge”

D. Some applications in bridge constructing

Overall view of the collapse of the three central spans of the bridge at Agua Caliente (Guatemala) caused by the 1976 earthquake







7. Viscoelastic dampers for vibration and seismic protection

7.5. Shock and vibration absorbers in civil and industrial engineering

7.5.1. Dashpots (dampers) for seismic protection

7.5.1.1. Dashpots by “Taylor Devices” and “Scott Forge”

E. Ultimate capacity of the dashpots

Capacity up to: 2,000,000 pounds (9072 KN)

Strokes of up to: 120 inches (3.048 m)

Temperatures: -40 ÷ +160 F (-40 ÷ +70 C)

35-year warranty

7. Viscoelastic dampers for vibration and seismic protection

7.5. Shock and vibration absorbers in civil and industrial engineering

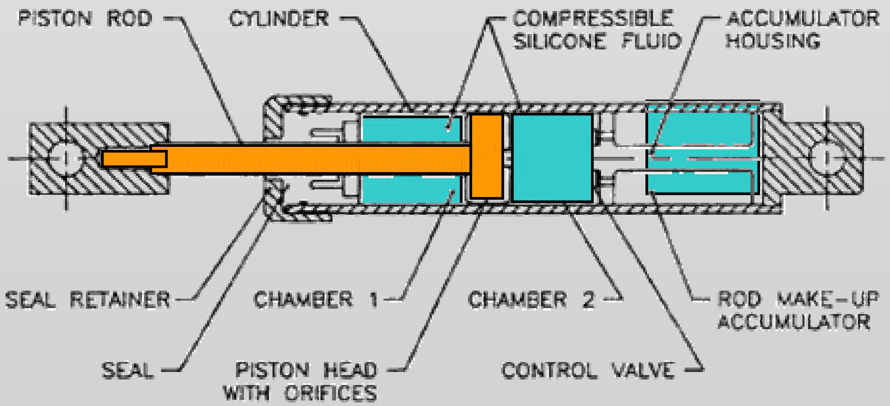
7.5.1. Dashpots (dampers) for seismic protection

7.5.1.1. Dashpots by “Taylor Devices” and “Scott Forge”

F. Concluding remark

The “Taylor Devices” position their dashpots, as dampers, i.e. the systems composed of dashpots + springs (Kelvin elements).

Possible explanation



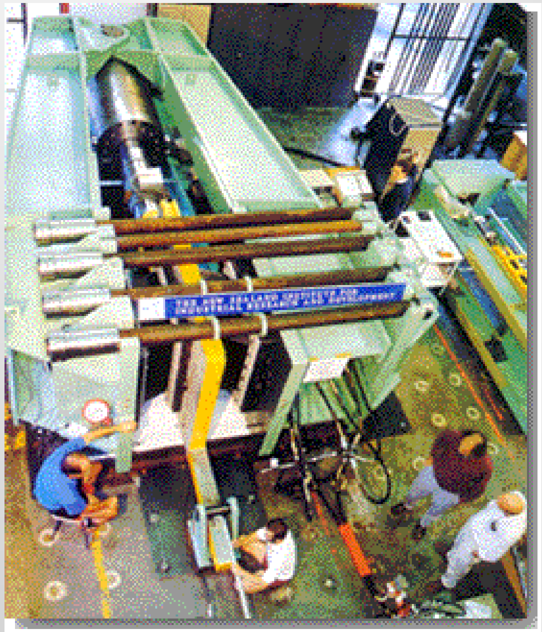
Valves in the piston!

7. Viscoelastic dampers for vibration and seismic protection

7.5. Shock and vibration absorbers in civil and industrial engineering

7.5.1. Dashpots (dampers) for seismic protection

7.5.1.2. Dashpots by “Robinson Seismic Limited”



Two bearings, each weighing three-quarters of a tonne, are put through a seismic-simulation test rig

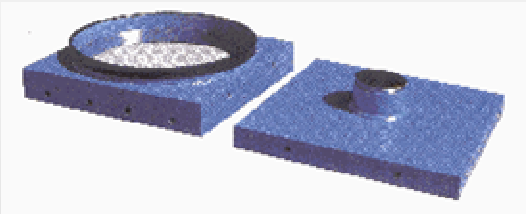
A bridge in Wellington, New Zealand with dashpots

7. Viscoelastic dampers for vibration and seismic protection

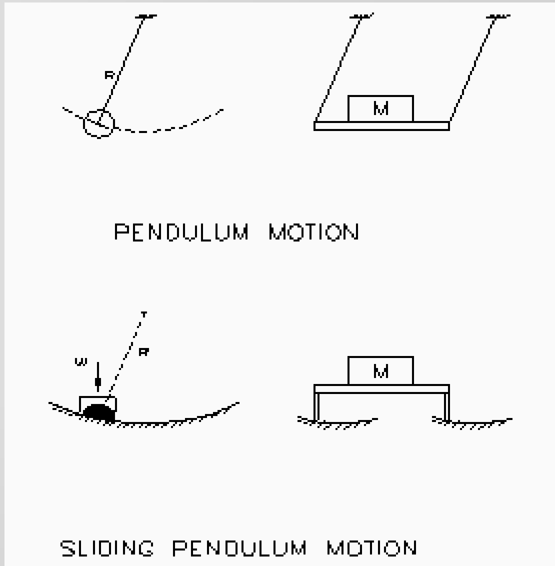
7.5. Shock and vibration absorbers in civil and industrial engineering

7.5.2. Friction Pendulum Seismic Isolation Bearings

7.5.2.1. The main principle



Bearing Components



Basic Principles

By the “Earthquake Protection Systems, Inc.”, CA, USA

EPS Inc.

7. Viscoelastic dampers for vibration and seismic protection

7.5. Shock and vibration absorbers in civil and industrial engineering

7.5.2. Friction Pendulum Seismic Isolation Bearings

7.5.2.2. Examples in civil engineering

San Francisco International Airport Terminal

by EPS Inc.

US Court of Appeals

7. Viscoelastic dampers for vibration and seismic protection

7.5. Shock and vibration absorbers in civil and industrial engineering

7.5.2. Friction Pendulum Seismic Isolation Bearings

7.5.2.3. Example in bridge construction

Friction Pendulum Bearing in the "American River Bridge" at Lake Natoma in Folsom (CA)

by EPS Inc.

7. Viscoelastic dampers for vibration and seismic protection

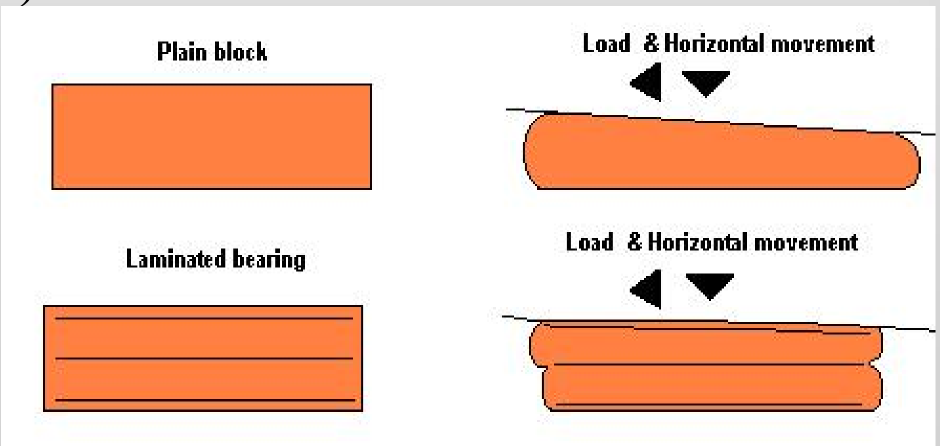
7.5. Shock and vibration absorbers in civil and industrial engineering

7.5.3. Elastomeric dampers

7.5.3.1. Examples of design

Plane rubber (neoprene) mats

Laminated metal (lead)-rubber mats



7. Viscoelastic dampers for vibration and seismic protection

7.5. Shock and vibration absorbers in civil and industrial engineering

7.5.3. Elastomeric dampers

7.5.3.2. Examples in civil and industrial engineering

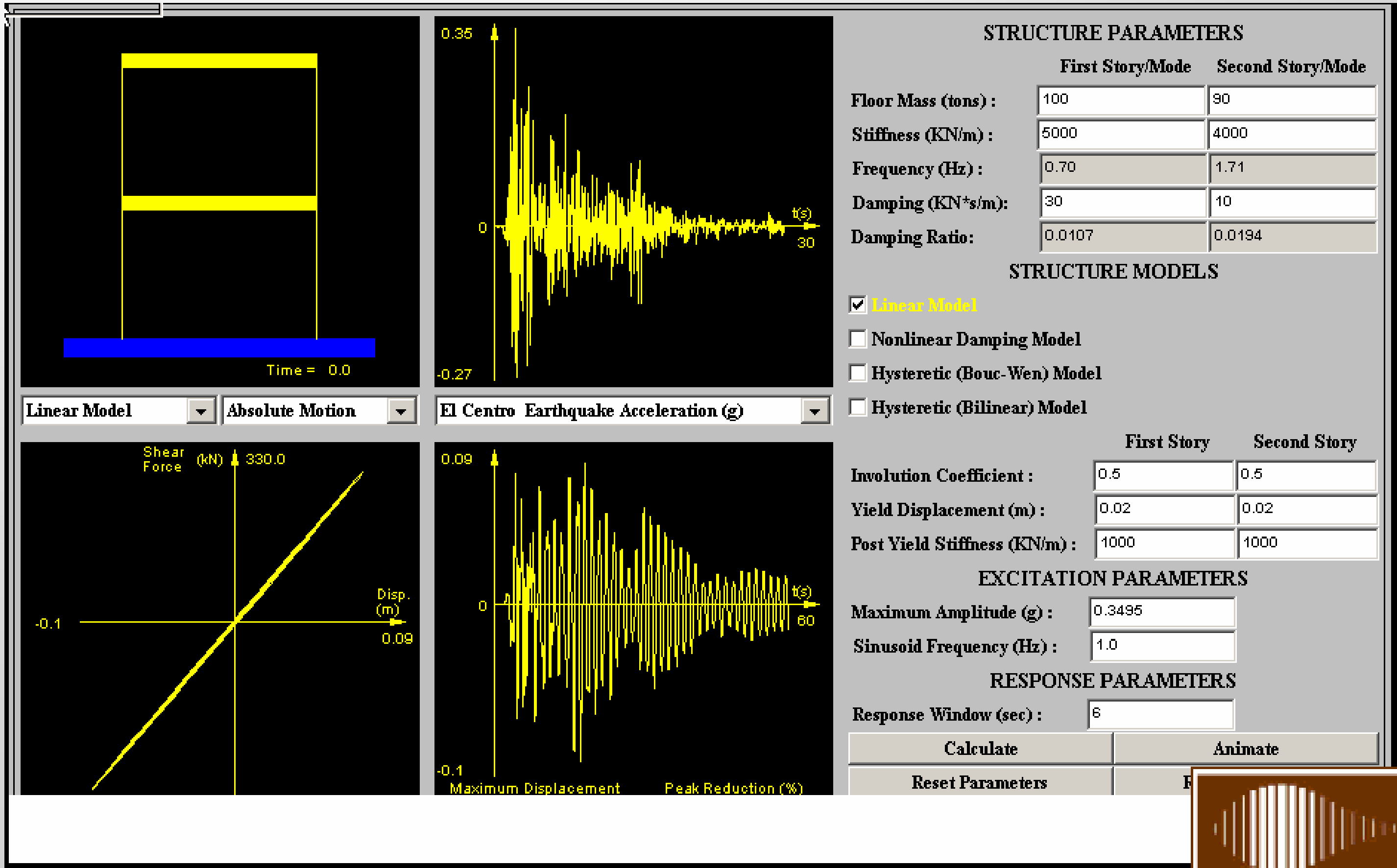
Laminated Neoprene mats for seismic protection by AARP (UAE)

Solid neoprene mat for column bearing by Agom International srl (Italy)

7. Viscoelastic dampers for vibration and seismic protection

7.5. Shock and vibration absorbers in civil and industrial engineering

7.5.4. Software of analyzing vibrations



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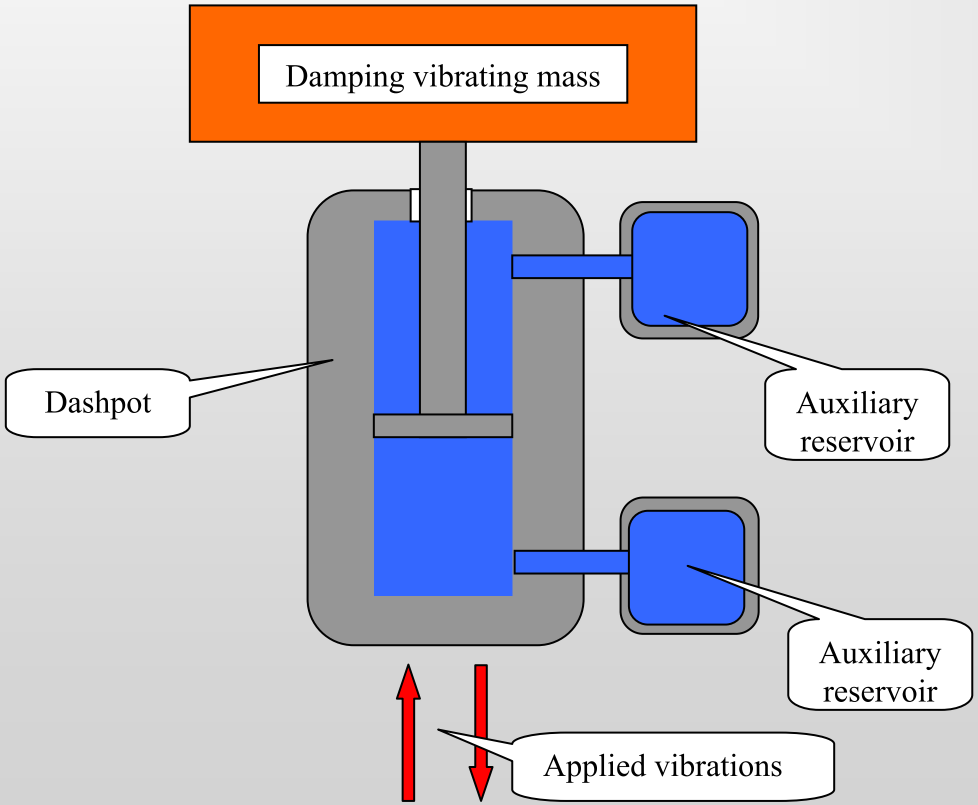
MULTIDISCIPLINARY CENTER FOR EARTHQUAKE ENGINEERING RESEARCH

7. Viscoelastic dampers for vibration and seismic protection

7.6. Active shock absorbers

7.6.1. Vertical arrangement

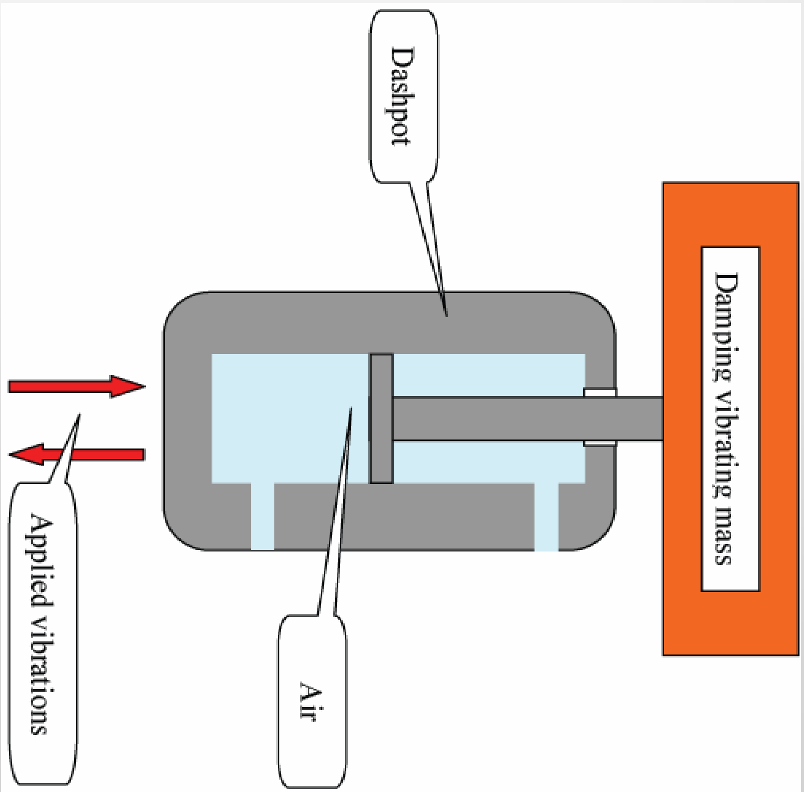


The main idea is to maintain the constant force acting on the upper mass to eliminate its vibrations, by using the so called “active” damping system

7. Viscoelastic dampers for vibration and seismic protection

7.6. Active shock absorbers

7.6.2. Horizontal arrangement

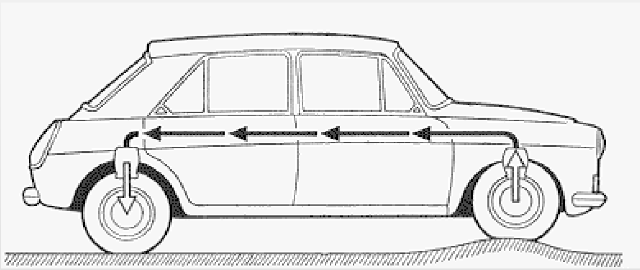


In the horizontal arrangement there is now need in maintaining constant liquid or gas presser in the chambers. The strut will go freely.

7. Viscoelastic dampers for vibration and seismic protection

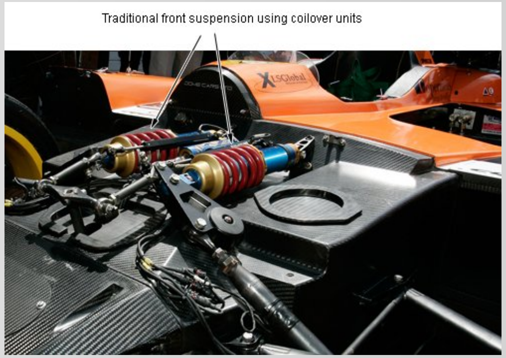
7.6. Active shock absorbers

7.6.3. Analogy with the automotive industry



An example of "hydrolastic" suspension proposed for Mini by Dr. Alex Moulton

Example of electronically controlled (electromotor driven) suspension by Bose Co.



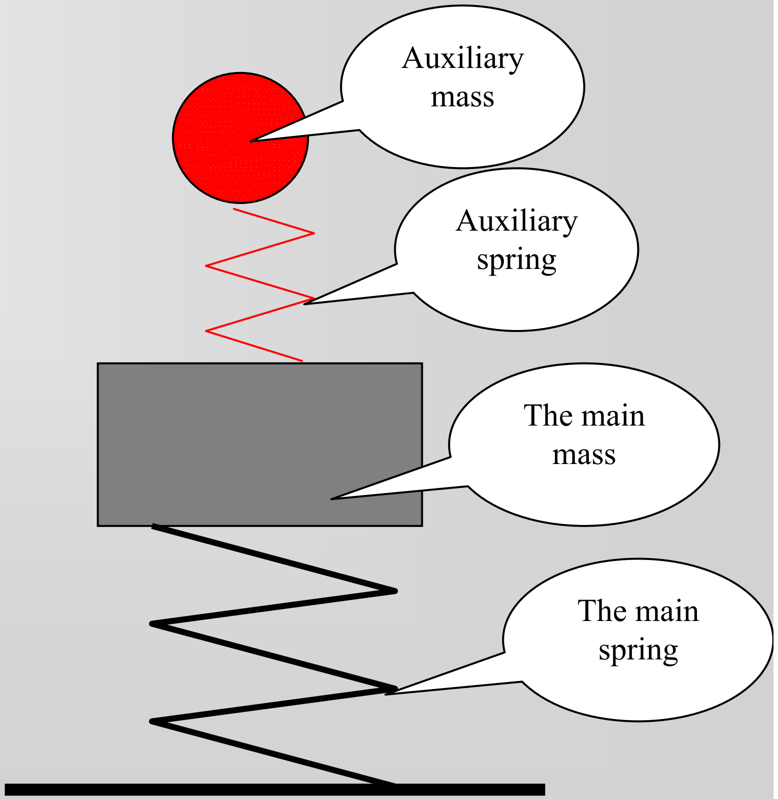
An example of horizontal displacement of dampers by "Racing for Holland" Co.

7. Viscoelastic dampers for vibration and seismic protection

7.7. Another principle of vibration absorbing

7.7.1. The basic idea

Damping vibrations can also be achieved by applying to the vibrating mass some additional mass connected with the first one with a suitable spring element:



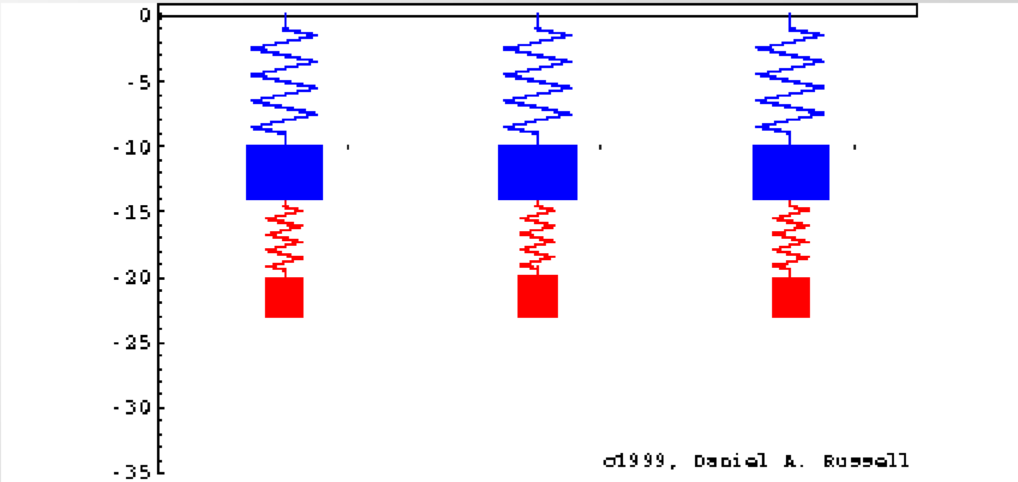
Remarks

- Such systems are known as the 2 DOF.
- They lead to a coupled system of two ODE.
- This principle was suggested by J. Ormondroyd and J.P. Den Hartog in 1928

7. Viscoelastic dampers for vibration and seismic protection

7.7. Another principle of vibration absorbing

7.7.2. Visualization



Remarks

The oscillating force is applied to the main mass

The left system vibrates with a frequency $0.67\omega_0$

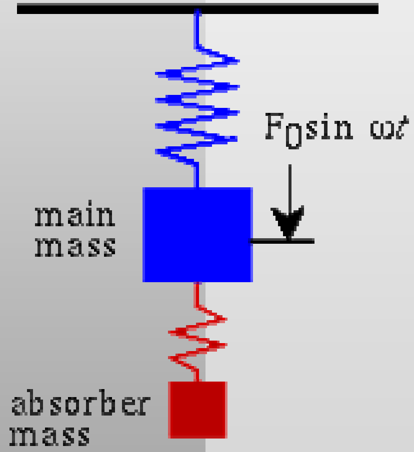
The middle system vibrates with the resonance frequency ω_0!

The right system vibrates with the frequency $1.3\ \omega_0$.

7. Viscoelastic dampers for vibration and seismic protection

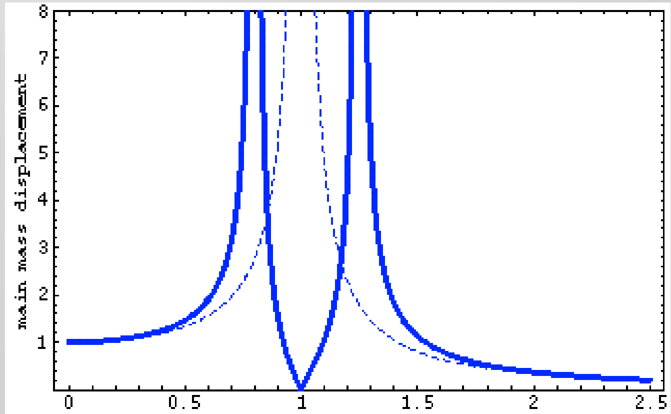
7.7. Another principle of vibration absorbing

7.7.3. The main equations



Coupled system of two ODE

$$\begin{cases} m_2\ddot{x}_2 + k_2 x_2 - k_1(x_1 - x_2) = F_0 \sin(\omega t) \\ m_1\ddot{x}_1 + k_1(x_1 - x_2) = 0 \end{cases}$$



8. Some hints on preventing appearing shock waves at building sites

8.1. Some observational data

Building site (Moscow, 2003)

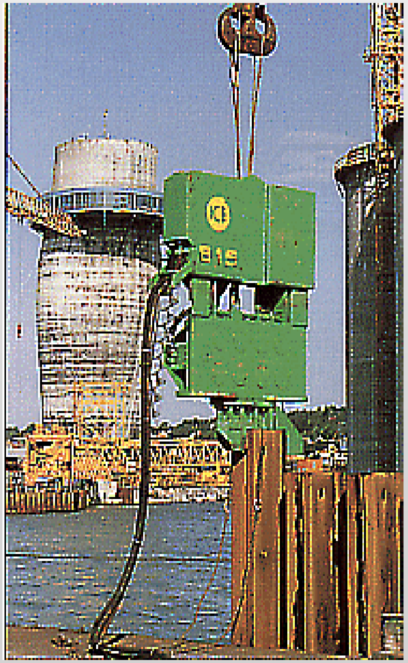
Cracks in an apartment house

8. Some hints on preventing appearing shock waves at building sites

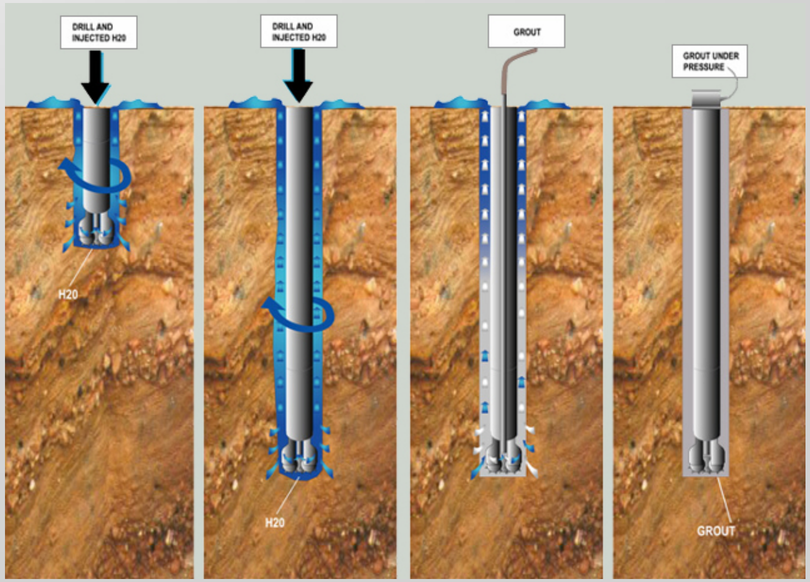
8.2. Techniques for eliminating shock waves at building sites

Diesel hummer for driving piles

Vibratory hummer for driving piles



Examples of the drilled piles (by “LayneGeo”)



Part IV. Review and conclusions

9. Concluding remarks

9.1. Brief summary

9.1.1. Seismic waves the main properties

Frequency range:	0.001 – 120Hz (1 – 70Hz are the most dangerous)
Speed range:	100 – 6500 m/sec
Wavelength range:	2 – 6500 m

9. Concluding remarks

9.1. Brief summary

9.1.2. Seismic waves scales

Richter magnitude test scale:

Logarithmic scale (0 – 10)

The modified Mercalli intensity scale:

Scale of intensity in grades (I – XII)

9. Concluding remarks

9.1. Brief summary

9.1.3. Bulk waves the main properties

The main theorem:

1. For any direction in an arbitrary anisotropic medium, there are three bulk waves:
   One (quasi) longitudinal and two (quasi) transverse waves
2. These waves can travel with (generally) different speeds
3. All bulk waves are not dispersing (wave speed does not depend upon frequency)

9. Concluding remarks

9.1. Brief summary

9.1.4. Surface acoustic waves classification

There are the following principle types of SAW:

1. **Rayleigh** waves (propagate in a halfspace)
2. **Stoneley** waves (propagate on an interface between two halfspaces)
3. **Love** waves (propagate in a layer and a halfspaces, have SH polarization)
4. **Lamb** waves (propagate in a layer)
5. **SH** waves (propagate in a layer, have H polarization)

9. Concluding remarks

9.1. Brief summary

9.1.5. The main principles of deterministic predicting analysis

1. Approximating using some analytical bases functions
2. Obtaining the desired extrapolation

Remark

Presumably, the best suited for the functions with the unknown periodicity and relatively short-time predictions are interpolating (Lagrange or Newton) polynomials, provided computations are done with the multiprecision arithmetic

9. Concluding remarks

9.1. Brief summary

9.1.6. The main principles of seismic wave protection

1. Modifying surface layers
   a. Creating “rough” surfaces
   b. Modifying physical properties of the surface layer
2. Creating seismic barriers
   a. Barriers reflecting seismic waves
   b. Barriers scattering wave energy
3. Installing dampers
   a. Discrete dampers composed of a dashpot and a spring
   b. Continuous type viscoelastic dampers (pads)

9. Concluding remarks

9.2. Directions for further studies

9.2.1. Theoretical studies of surface acoustic waves (analytical methods);

9.2.2. Interaction of bulk or surface waves with barriers (mainly FEM);

9.2.3. Scattering of elastic waves by inclusions and creating wave scattering barriers (mainly FEM);

9.2.4. Vibro- and seismic damper engineering (mainly ODE);

9.2.5. Methods and algorithms for predicting (mainly numerical methods).

9. Concluding remarks

9.3. The concluding note

9. Concluding remarks

9.4. Recommended literature

9.4.1. Earth structure

Van der Pluijm Ben A. and Marshak Stephen,
Earth structure,
McGraw-Hill, 2002, ISBN: 0697172341

Pollard David D., Fletcher Raymond C.,
Fundamentals of Structural Geology,
Cambridge Univ. Press, 2005, ISBN: 10- 052183927

9. Concluding remarks

9.4. Recommended literature

9.4.2. Experimental methods in geophysics

D.J. Rockhill, D.J. White, and M.D. Bolton,
Ground vibrations due to piling operations,
BGA Int. Conf. on Foundations, 2003, Dundee, Scotland, 743-756

Stephen E. Prensky,
A Survey of Recent Developments and Emerging Technology in Well Logging and Rock Characterization,
The Log Analyst, 1994, v. 35, N.2, p.15-45, N.5, p.78-84 (extensive bibliography)

9. Concluding remarks

9.4. Recommended literature

9.4.3. Theory of anisotropic elasticity

Rand Omri and Rovenski Vladimir Y.
Analytical Methods in Anisotropic Elasticity with Symbolic Computational Tools,
Birkhäuser, 2005, 451 p., ISBN-10: 0-8176-4272-2

Ting T.C.T.,
Anisotropic Elasticity: Theory and Applications,
Oxford Univ. Press, 1996, ISBN-13: 9780195074475

Lekhnitskii S. G.,
Theory of Elasticity of an Anisotropic Body,
Holden-Day, San Francisco, 1963.

9. Concluding remarks

9.4. Recommended literature

9.4.4. Theory of acoustic elastic waves

Auld B.A.,
Acoustic Fields and Waves in Solids,
2nd edition, Krieger Pub Co, 1990, ISBN: 0894644904

Kaufman A.A. and Levshin A.L.,
Acoustic and Elastic Wave Fields in Geophysics,
Parts I - III, Elsevier, 2000, ISBN: 0-444-50336-6

Kennett B. L. N.,
Seismic Wave Propagation in Stratified Media,
Cambridge University Press; Reprint edition, 1985, ISBN: 0521312191

9. Concluding remarks

9.4. Recommended literature

9.4.5. Engineering methods in protecting from seismic waves

Hansbo S.,
Foundation Engineering,
Elsevier, 1994, ISBN-13: 978-0-444-88549-4

Earthquake Proof Design and Active Faults,
Editor Y. Kanaori, Elsevier, 1997, ISBN-13: 978-0-444-82562-9

Mahtab M.A. and Grasso P.,
Geomechanics Principles in the Design of Tunnels and Caverns in Rocks,
Elsevier, 1992, ISBN-13: 978-0-444-88308-7