Structure of the presentation

Part I

A brief description of the Institute for Problems in Mechanics (IPM) of Russian Academy of Sciences

Part II

An introduction to terminology and basic problems in the Surface Acoustic Wave (SAW) studies

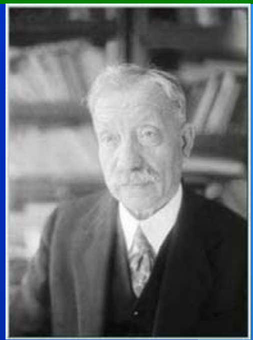
Part III

Some recent results in SH- and Love wave analyses

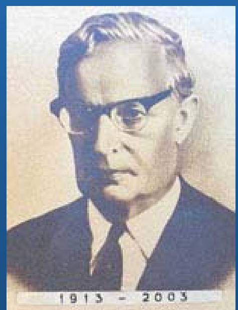
Part I

Institute for Problems in Mechanics
of
Russian Academy of Sciences

History of the IPM



The first director of the former Institute of Mechanics Professor **B.G. Galerkin** ~1938



The first director of the Institute for Problems in Mechanics of Russian Academy of Sciences **A.Y. Ishlinskiy** 1965-1990

History of the IPM

The following persons worked in IPM in different years

Professor L.A. Goldenveizer (theory of shells, asymptotic methods)

Professor N.V. Zvolinskij (theory of surface waves, analysis of tsunami)

Professor G.I. Barenblatt (fracture mechanics, hydrodynamics)

Professor O.A. Olejnik (differential equations, homogenization)

IPM

Laboratory of Wave Dynamics

The head of Laboratory Professor I.V. Simonov

Fracture mechanics and Astromechanics

IPM

Laboratory of Wave Dynamics

The main directions:

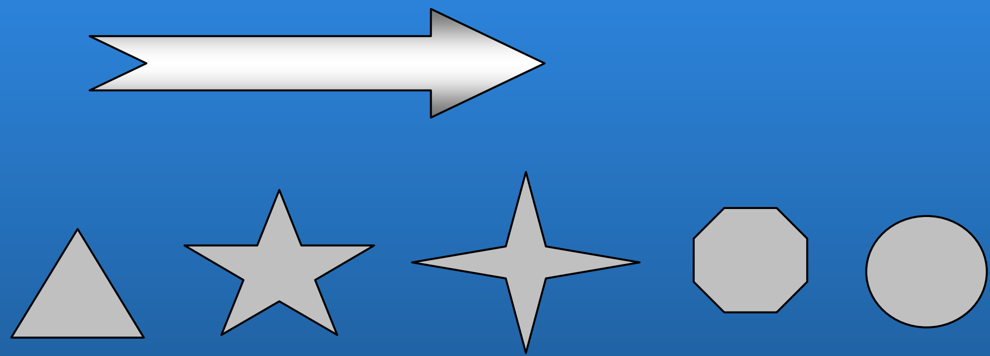
Analyses of bodies collisions and aftereffects:
Fragmentation, Cavern-and-crack formation, Penetration

IPM

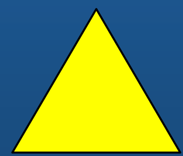
Laboratory of Wave Dynamics

Experimental study:

Penetrating ability of different arrow-heads



The winner with respect to the penetrating ability



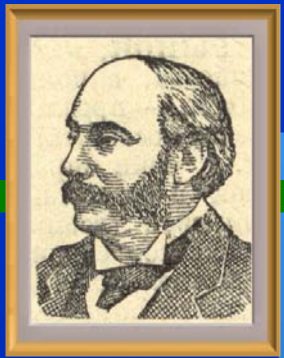
Part II

Introduction to the Surface Acoustic Wave analyses

The main types of Surface Acoustic Waves

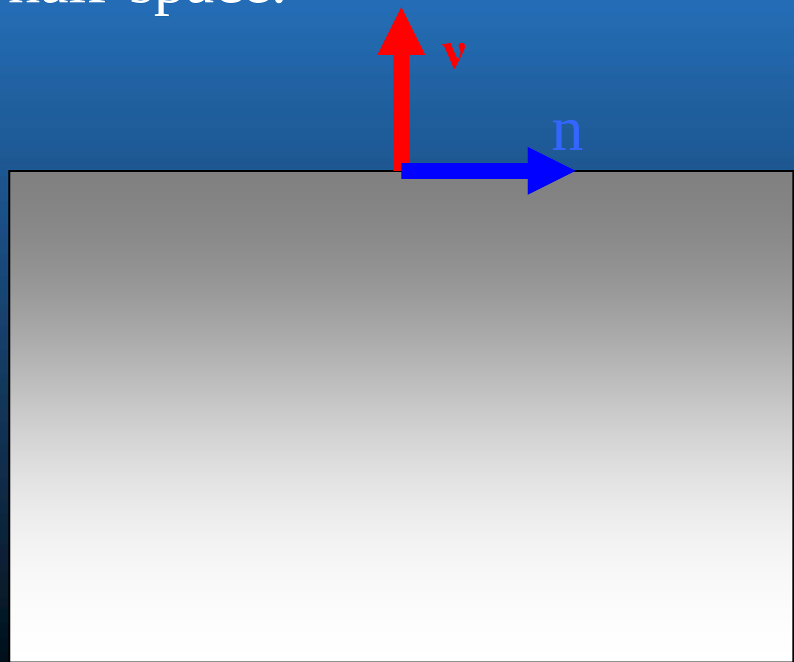
Rayleigh waves

Basic definitions

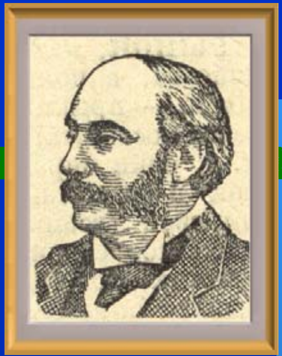


Pioneering Rayleigh's work (1885) where these waves were described.

Rayleigh surface wave means attenuating with depth elastic wave with a plane wave front propagating on a traction-free boundary of a half-space.

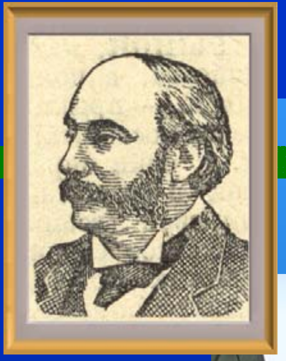


Rayleigh waves,
their role in seismic activity

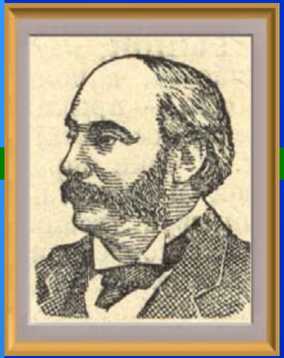


These waves play a very important role in transmitting the seismic energy and causing the catastrophic destructions due to the seismic activity.

Rayleigh waves,
danger for structures



Rayleigh waves,
Comparing different seismic waves



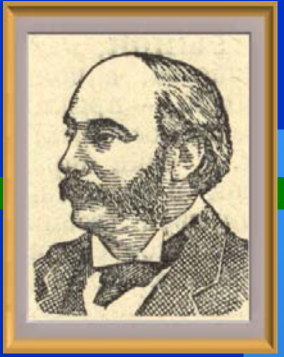
1. Speed of propagation (phase speed)

$$c_{Love} < c_{Rayleigh} < c_{bulk}^{Transverse} < c_{bulk}^{Longitudinal}$$

2. Transmitted elastic energy

$$E_{Love} \sim E_{Rayleigh} >> E_{bulk}^{Transverse} \sim E_{bulk}^{Longitudinal}$$

Rayleigh waves



Problem of finding the secular equation

For an isotropic medium the secular equation (Rayleigh equation) is:

$$(\lambda+2\mu)(\lambda+\mu)x^3-8\mu(\lambda+2\mu)(\lambda+\mu)x^2+$$

$$+8\mu^2(\lambda+\mu)(3\lambda+4\mu)x-16\mu^3(\lambda+\mu)^2=0$$

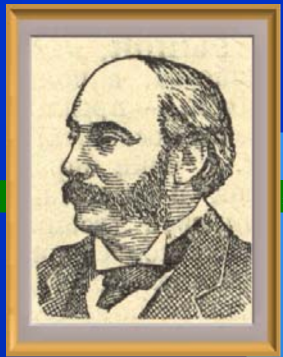
where

$$x=\rho c^2$$

What is the secular equation for an anisotropic medium?

Rayleigh waves

Problem of finding the secular equation



For an arbitrary anisotropic medium the secular equation has not been found yet (2006)

Currie,	1979
Taziev,	1989
Destrade,	2001
Ting,	2002, 2003

Related problem

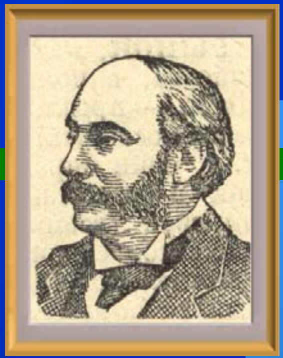
The related problem: How many independent constants have an arbitrary anisotropic elasticity tensor?

$$\mathbf{C}_{6\times6} = \mathbf{Q}^t{}_{6\times6} \cdot \begin{pmatrix} d_2 & & & & & \\ & d_2 & & & & \\ & & d_3 & & & \\ & & & d_4 & & \\ & & & & d_5 & \\ & & & & & d_6 \end{pmatrix} \cdot \mathbf{Q}_{6\times6}$$

The new problem:
How many independent constants has a rotation in *R*^{6}?

Rayleigh waves

The main practical problem

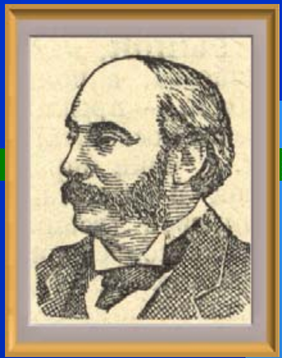


For a long time the main problem related to Rayleigh wave propagating on an **arbitrary anisotropic** half-space was finding conditions at which such a wave cannot propagate (Problem of “forbidden” directions).

Do “forbidden directions” exist?

Rayeleigh waves

The main practical problem



Theorem of existence for Rayleigh waves:

Barnett and Lothe,	1973-76
Chadwick,	1975-85
Ting,	1983-96

In 1998-2002 a type of Non-Rayleigh waves was observed and constructed explicitly.

Such a wave corresponds to appearing the **Jordan blocks** in a six-dimensional matrix associated with the Christoffel equation.

A remark on Jordan blocks appearing in the matrix analysis

Marie Ennemond Camille

Jordan

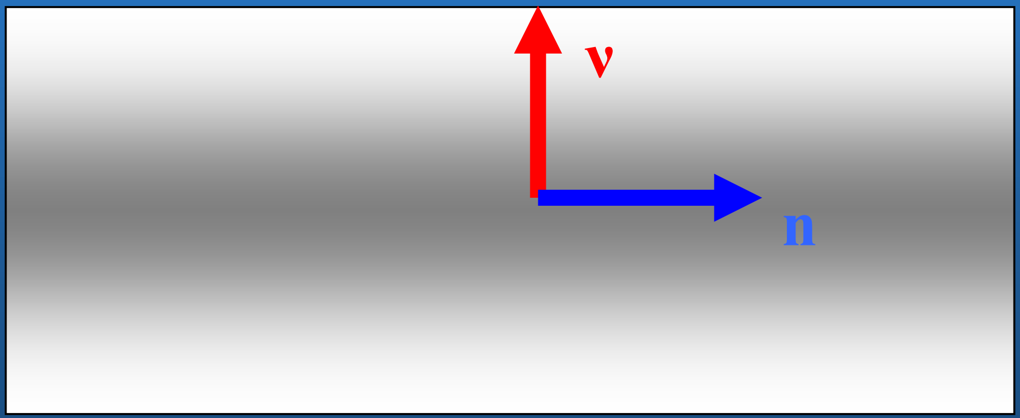
1838 - 1922

3/13/2009

Lamb waves

Basic definitions

These waves, discovered by Horace Lamb (1917), can propagate in a layer with either traction-free, clamped or mixed boundary conditions, imposed on the outer surfaces of a layer.



Lamb waves

Basic properties

In contrast to Rayleigh waves, Lamb waves are highly dispersive, that means the the phase speed depends upon frequency or wavelength.

There can be an infinite number of Lamb waves propagating with the same phase speed and differing by the frequency.

Lamb waves can travel with both sub, intermediate, and supersonic speed.

An interesting physical observation:

After excitation, the most energy is transferred by the two lowest modes (symmetric and flexural).

Lamb waves

The main problems

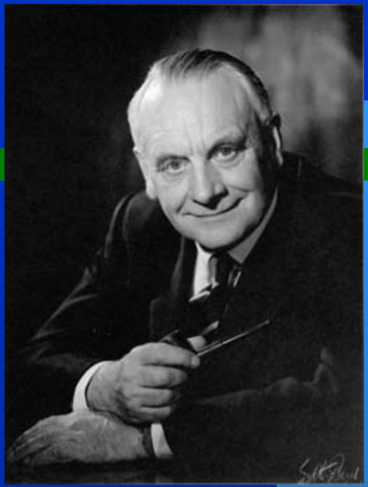
Presumably, the main problem related to the genuine Lamb wave propagation lies in constructing solutions for an anisotropic layer having arbitrary elastic anisotropy:

Deriving explicit secular equation(s) for the phase speed and frequency (for different speed intervals there may be needed different equations)

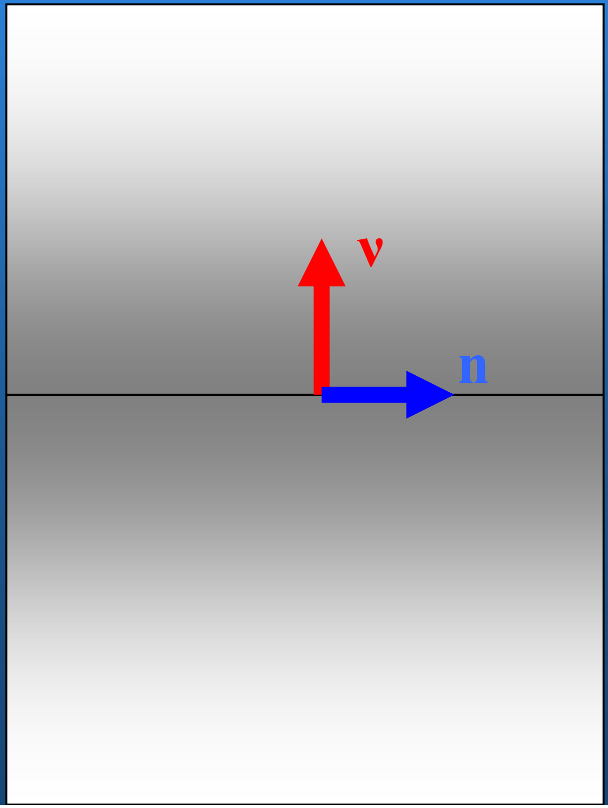
Obtaining and analyzing solutions for waves related to appearing the Jordan blocks in a six-dimensional matrix formalism.

Stoneley waves

Definition



These waves were described by Robert Stoneley (1924), and they are waves traveling on an interface between two contacting half-spaces.



Stoneley waves

Basic properties

Stoneley waves exponentially attenuate with depth in both half-spaces, and in this respect resemble Rayleigh waves.

Both Rayleigh and Stoneley waves are not dispersive.

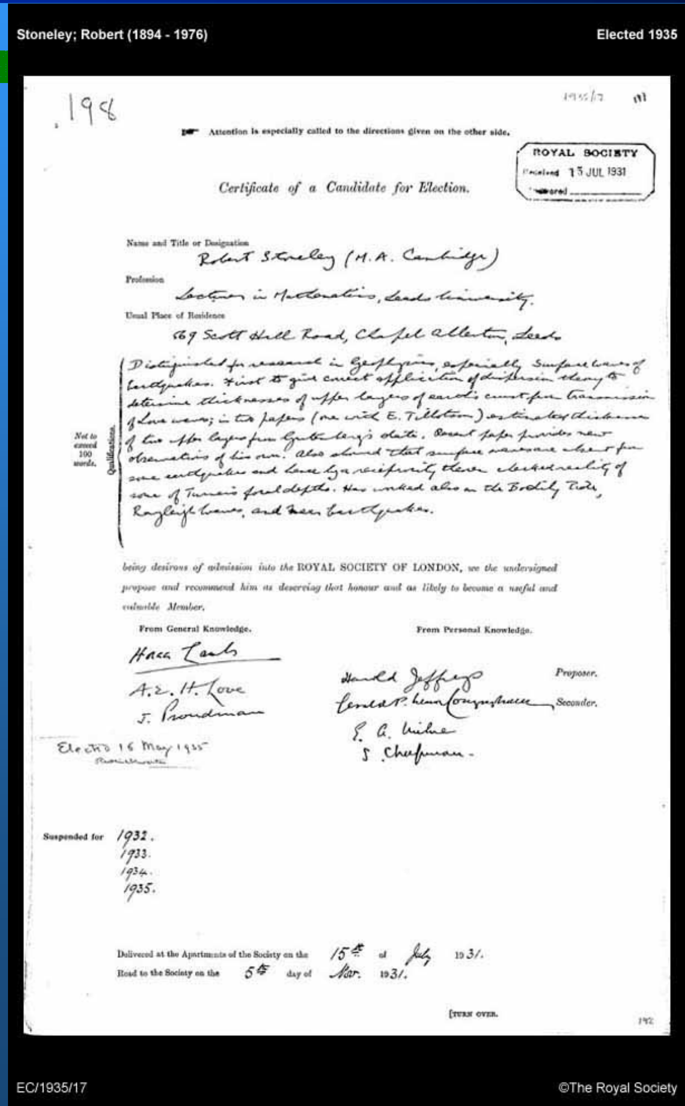
Remark

For contacting isotropic half-spaces conditions of existence were found by Stoneley.

Conditions for existence of Stoneley waves propagating in anisotropic half-spaces are not established (2006).

Robert Stoneley

A fact from biography



Stoneley; Robert (1894 - 1976) Elected 1935

198

1935/17

Attention is especially called to the directions given on the other side.

ROYAL SOCIETY
Received 13 JUL 1931

Certificate of a Candidate for Election.

Name and Title or Designation
Robert Stoneley (M.A. Cambridge)

Profession
Lecturer in Mathematics, Leeds University.

Usual Place of Residence
569 Scott Hall Road, Chapel Allerton, Leeds

Qualifications. (Not to exceed 100 words.)
Distinguished for research in Geophysics, especially Surface waves of Earthquakes. First to give correct application of dispersion theory to determine thicknesses of upper layers of earth's crust from transmission of Love waves; in two papers (one with E. Tillotson) estimates thickness of the upper layers from Gutenberg's data. Recent paper provides new observations of his own. Also showed that surface waves are absent for some earthquakes and hence by a reciprocity theorem checked reality of some of Turner's focal depths. Has worked also on the Bodily Tide, Rayleigh waves, and near earthquakes.

being desirous of admission into the ROYAL SOCIETY OF LONDON, *we the undersigned propose and recommend him as deserving that honour and as likely to become a useful and valuable Member.*

From General Knowledge.
Horace Lamb
A. E. H. Love
J. Proudman

From Personal Knowledge.
Harold Jeffreys — Proposer.
Gerald P. Lenox-Conyngham — Seconder.
E. A. Milne
S. Chapman.

Elected 16 May 1935

Suspended for 1932. 1933. 1934. 1935.

Delivered at the Apartments of the Society on the 15th of July 1931.
Read to the Society on the 5th day of Nov. 1931.

[TURN OVER.

EC/1935/17

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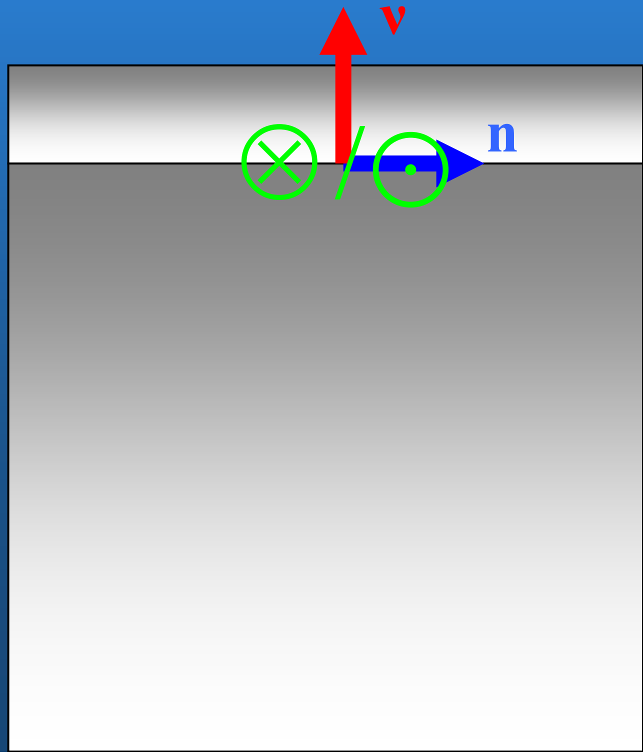
Love waves

Definition

These waves originate to Augustus Love (1911), who for the first time obtained a solution for a wave traveling in a system consisted of a layer and a contacting half-space.

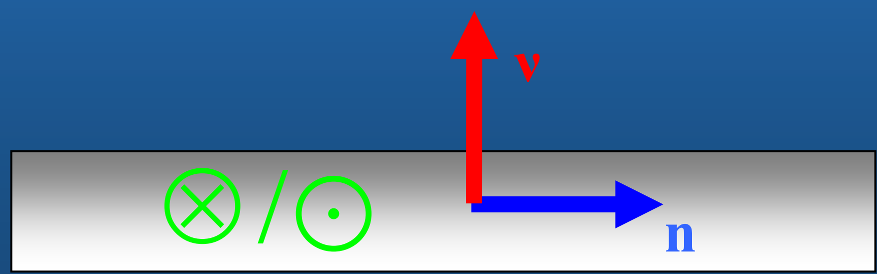


SH waves

Definition

No image available

These waves travel in a layer or possibly several contacting layers, and have the transverse horizontal polarization.



Part III

Love waves in stratified monoclinic media

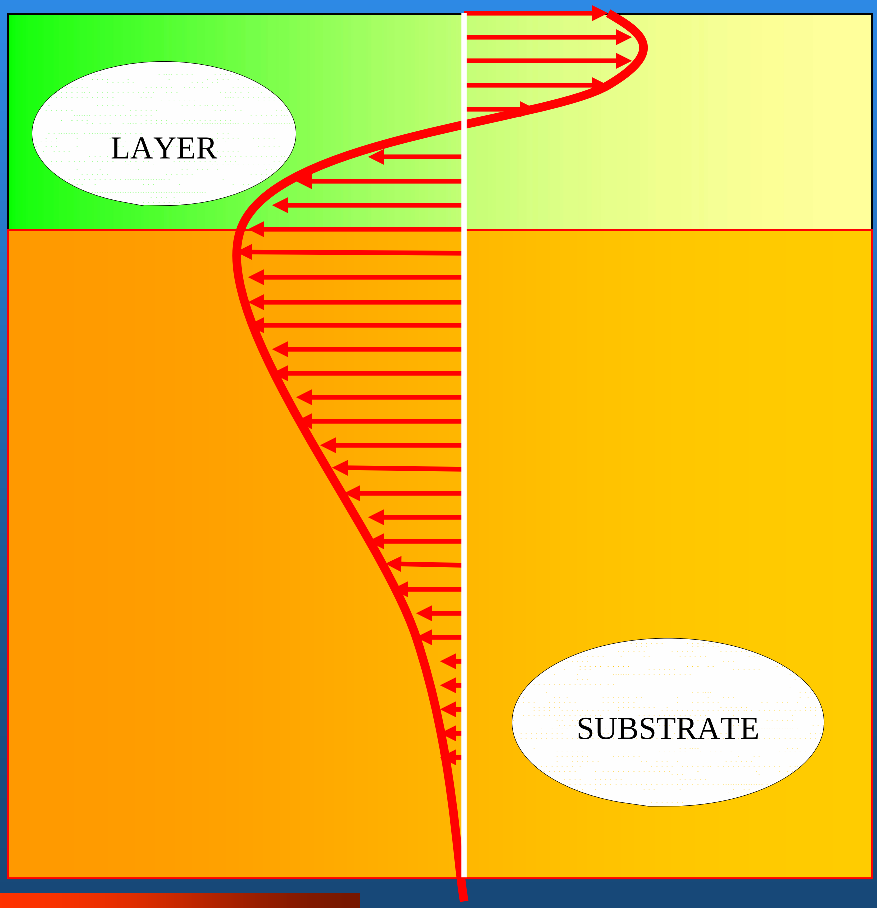
Main definition

Love wave is a surface wave having horizontal transverse polarization and propagating in a medium composed of an elastic layer lying on a substrate.

MORE INFO...

It is assumed that Love wave attenuates with depth in a substrate.

Love waves: polarization

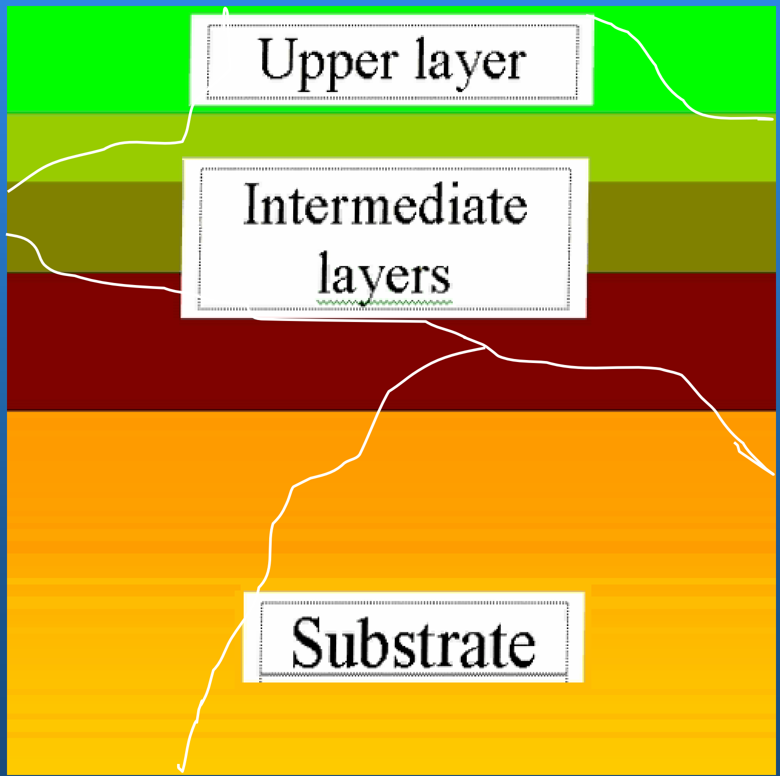


Basic theoretical works

Love	**1911**	**Representation for surface waves with SH-polarization, propagating in a system composed of an *isotropic* layer lying on *isotropic* substrate**
Thomson	**1950**	**The first analysis of waves in stratified media**
Haskell	**1953**	**Correction of the previous results, introduction of the Transfer Matrix Method**
Knopoff	**1964**	**Introduction of the Global Matrix Method**
Dieulesaint et Royer	**1980**	**Equations for speed and polarization of Love waves in *orthotropic* media**

Multilayered structure

Free surface



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The main problem of practical importance

Determining geometrical and physical properties of the internal layer(s) by analyzing the dispersion relations of Love surface waves propagating in the multi layered structure

Analysis scheme

I. Single layer

i. Obtaining Christoffel equation;

ii. Determining the Christoffel parameters

II. Multiple layers

i. Formulating contact type boundary conditions

ii. Obtaining the Global (Transfer) Matrix

III. Numerical implementation

Single layer analysis.

Equations of motion

$$\mathbf{A}(\partial_x, \partial_t)\mathbf{u} \equiv \operatorname{div}_x \mathbf{C} \cdot\cdot \nabla_x \mathbf{u} - \rho \ddot{\mathbf{u}} = 0$$

WHERE

$\mathbf{C}$ Is the elasticity tensor, assumed to be positive definite and hyper elastic;

$\mathbf{u}$ Is the displacement field;

ρ Is the material density;

Single layer analysis.

Monoclinic anisotropy

Definition:

The material is called monoclinic (with respect to a direction $\mathbf{m}$) if its symmetry group is generated by $\mathbf{R}_{\mathbf{m}}^{\pi}$

REMARKS

I. The definition is equivalent to vanishing all of the decomposable components of the tensor $\mathbf{C}$ having the odd number of entries of vector $\mathbf{m}$;

II. The assuming monoclinic symmetry provides a sufficient condition for the surface tractions acting on any plane ($\mathbf{v}\cdot\mathbf{x}=const$) to be collinear with vector $\mathbf{m}$;

III. The elasticity tensor for monoclinic medium has **13** independent elasticity constants

Single layer analysis.

Representation for displacements for Love wave

$$\mathbf{u}(\mathbf{x},t) \equiv \mathbf{m}\, f\,(ir\,x')\,e^{ir(\mathbf{n}\cdot\mathbf{x}-ct)}$$

WHERE

$\mathbf{m} = \nu \times \mathbf{n}$ is the amplitude vector ;

$\mathbf{n}$ is direction of propagation;

ν is the unit normal to the median plane;

$x' = \nu \cdot \mathbf{x}$

f is the unknown scalar function;

Single layer analysis.

The Christoffel equation

$$\left(\begin{matrix}(\mathbf{m}\otimes\boldsymbol{\nu}\cdot\cdot\mathbf{C}\cdot\cdot\boldsymbol{\nu}\otimes\mathbf{m})\partial_{x'}^2+(\mathbf{m}\otimes\boldsymbol{\nu}\cdot\cdot\mathbf{C}\cdot\cdot\mathbf{n}\otimes\mathbf{m}+\mathbf{m}\otimes\mathbf{n}\cdot\cdot\mathbf{C}\cdot\cdot\boldsymbol{\nu}\otimes\mathbf{m})\partial_{x'}+\\ (\mathbf{m}\otimes\mathbf{n}\cdot\cdot\mathbf{C}\cdot\cdot\mathbf{n}\otimes\mathbf{m}-\rho c^2)\end{matrix}\right)f(x')=0$$

The corresponding characteristic equation is:

$$\left(\begin{matrix}(\mathbf{m}\otimes\boldsymbol{\nu}\cdot\cdot\mathbf{C}\cdot\cdot\boldsymbol{\nu}\otimes\mathbf{m})\gamma^2+(\mathbf{m}\otimes\boldsymbol{\nu}\cdot\cdot\mathbf{C}\cdot\cdot\mathbf{n}\otimes\mathbf{m}+\mathbf{m}\otimes\mathbf{n}\cdot\cdot\mathbf{C}\cdot\cdot\boldsymbol{\nu}\otimes\mathbf{m})\gamma+\\ (\mathbf{m}\otimes\mathbf{n}\cdot\cdot\mathbf{C}\cdot\cdot\mathbf{n}\otimes\mathbf{m}-\rho c^2)\end{matrix}\right)=0$$

REMARK

Multiple roots arise when the dicriminant vanishes

Single layer analysis.
Representations for the solution

Aliquant roots

$$\mathbf{u}(\mathbf{x},t)=\mathbf{m}\left(C_1\sinh(ir\alpha x')+C_2\cosh(ir\alpha x')\right)e^{ir(\beta x'+\mathbf{n}\cdot\mathbf{x}-ct)}$$

WHERE

C_1, C_2 are arbitrary coefficients defined by boundary and interfacial conditions

Boundary conditions
Single layer on a substrate

I. Traction-free boundary conditions at the upper boundary

$$\mathbf{t}_{layer}(\frac{h}{2},t)=0$$

II. Contact type boundary conditions at the interface

$$\mathbf{u}_{layer}(-\frac{h}{2},t)=\mathbf{u}_{substrate}(0,t)$$

$$\mathbf{t}_{layer}(-\frac{h}{2},t)=\mathbf{t}_{substrate}(0,t)$$

III. Sommerield’s attenuation condition

$$\lim_{x'\to-\infty}\mathbf{u}_{substrate}(x',t)=0$$

Method of analysis for multiple layers on the substrate /Global Matrix Method/

Example: Four anisotropic layers lying on an anisotropic substrate.

The dispersion equation:

$$\det(M) = 0$$

MORE INFO...

$$M \equiv \begin{pmatrix} A_1^+ & & & & \\ A_1^- & A_2^+ & & & \\ & A_2^- & A_3^+ & & \\ & & A_3^- & A_4^+ & \\ & & & A_4^- & A_5 \end{pmatrix}$$

Some analytical results for Love waves (one orthotropic layer on the orthotropic substrate)

Proposition. a) No Love wave can propagate in a system composed of a single orthotropic layer lying on an orthotropic substrate, when multiple roots in the Christoffel equation for the layer arise;

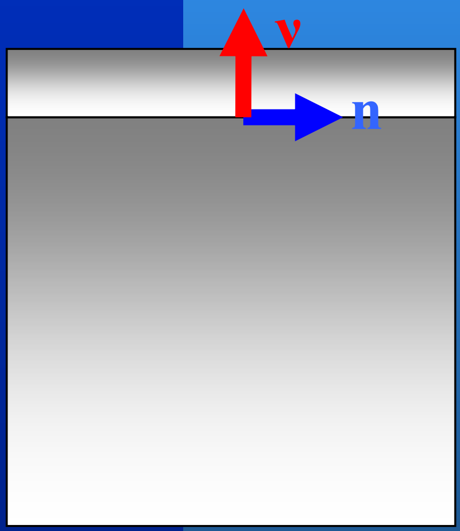
b) Love wave can propagate, if and only if the phase speed belongs to the interval

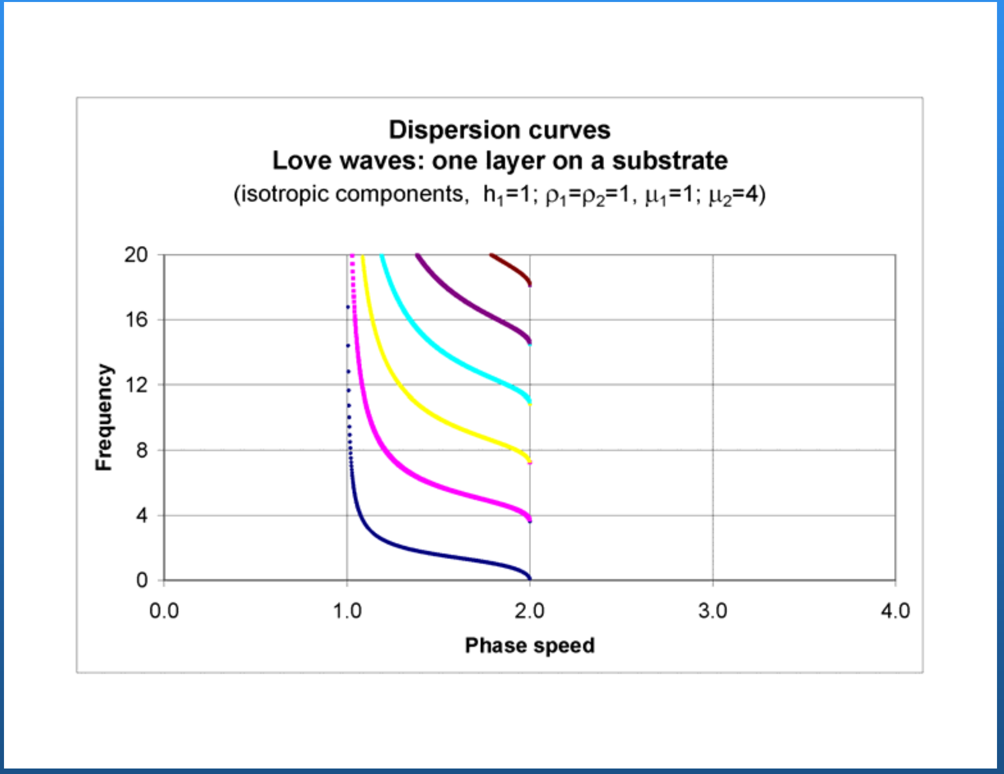
$c \in (c_{layer}^{bulk}\,;\ c_{substr}^{bulk});$

c) The dispersion relation admits the following representation:

$$\omega = \frac{c}{\gamma_1 h_1}\left(\arctan\left(i\,\frac{\gamma_2\left(\mathbf{m}\otimes\nu\cdot\cdot\mathbf{C}_2\cdot\cdot\nu\otimes\mathbf{m}\right)}{\gamma_1\left(\mathbf{m}\otimes\nu\cdot\cdot\mathbf{C}_1\cdot\cdot\nu\otimes\mathbf{m}\right)}\right)+n\pi\right), \qquad n = 0,1,2,\ldots;$$

Love waves (one layer on a substrate)





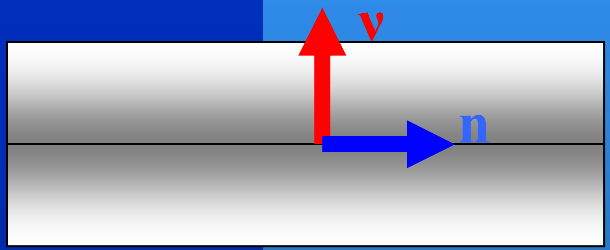
Remark

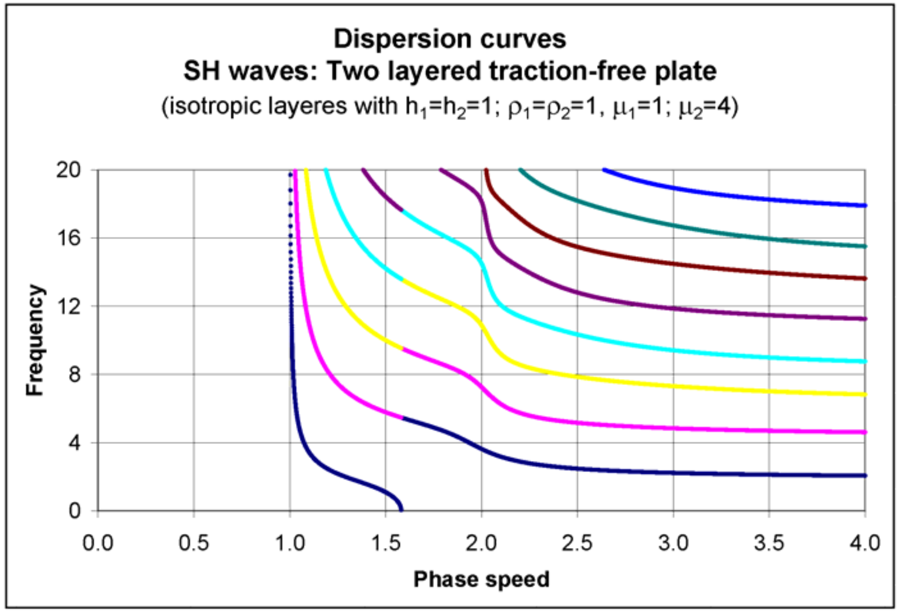
A layer should be less rigid, than a substrate. Otherwise, Love waves do not exist.

Numerical Example

SH waves

(two layered plate)

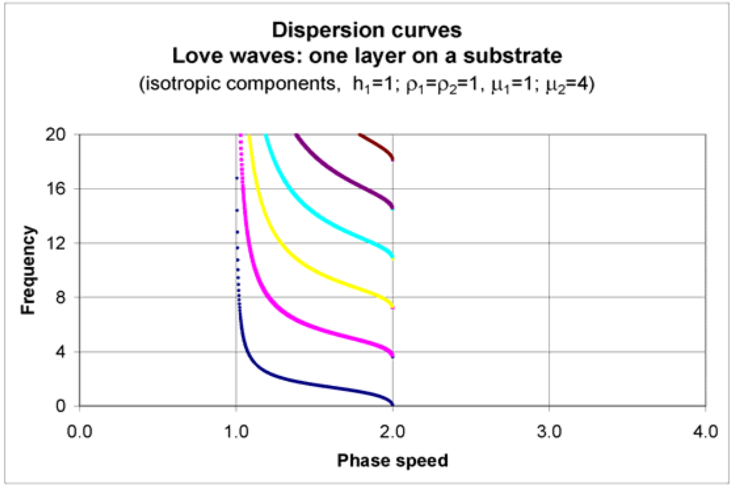


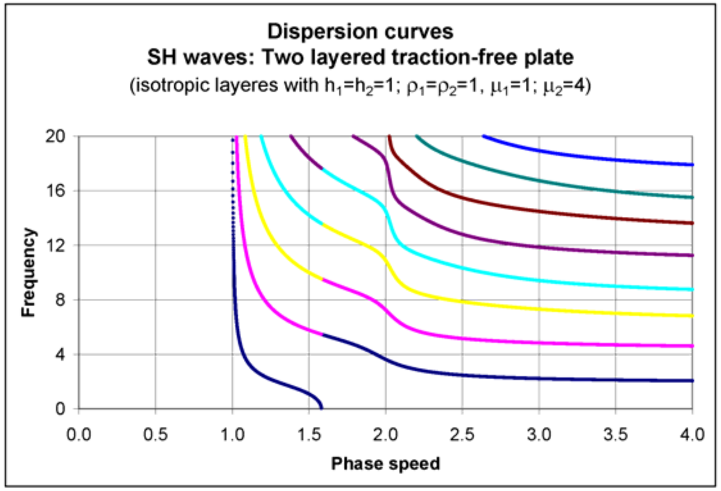


Remark

A lower mode wave near vanishing frequency is a soliton-like wave

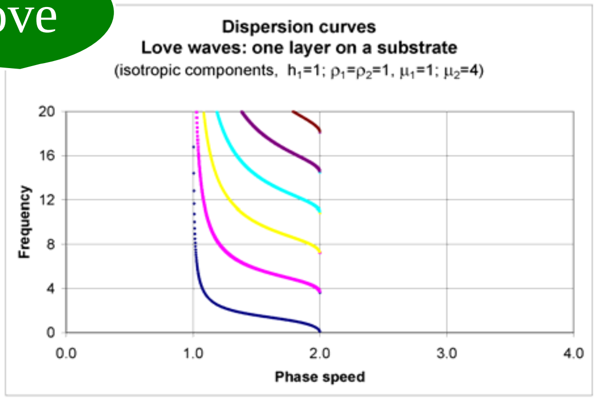
Love and SH waves (comparison)

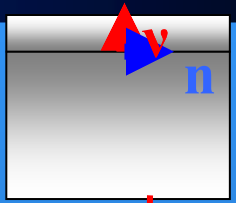




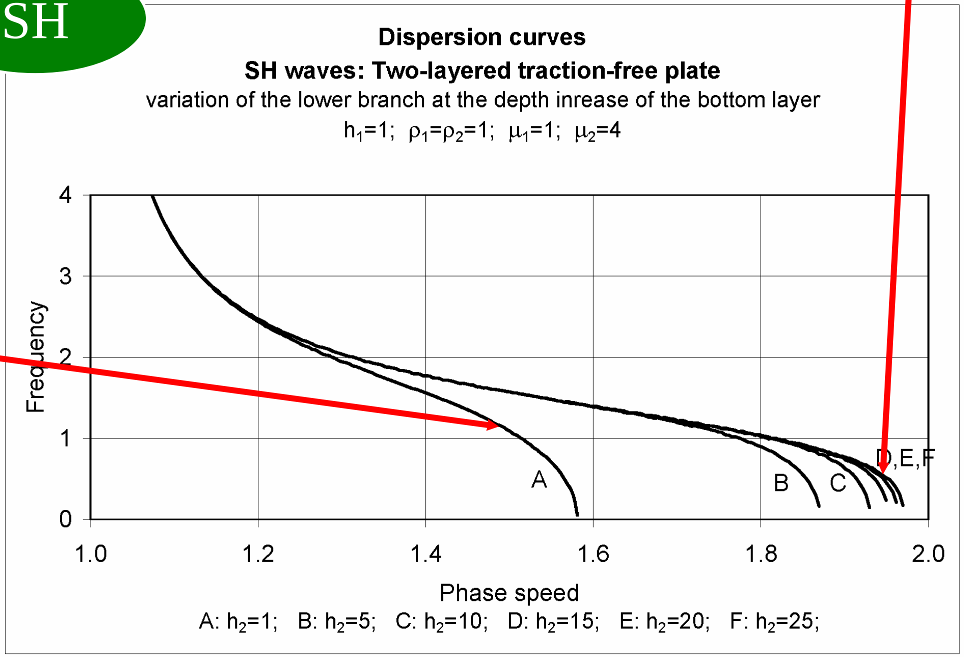
Love and SH waves (comparison)

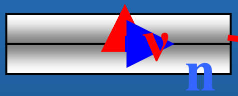
Love



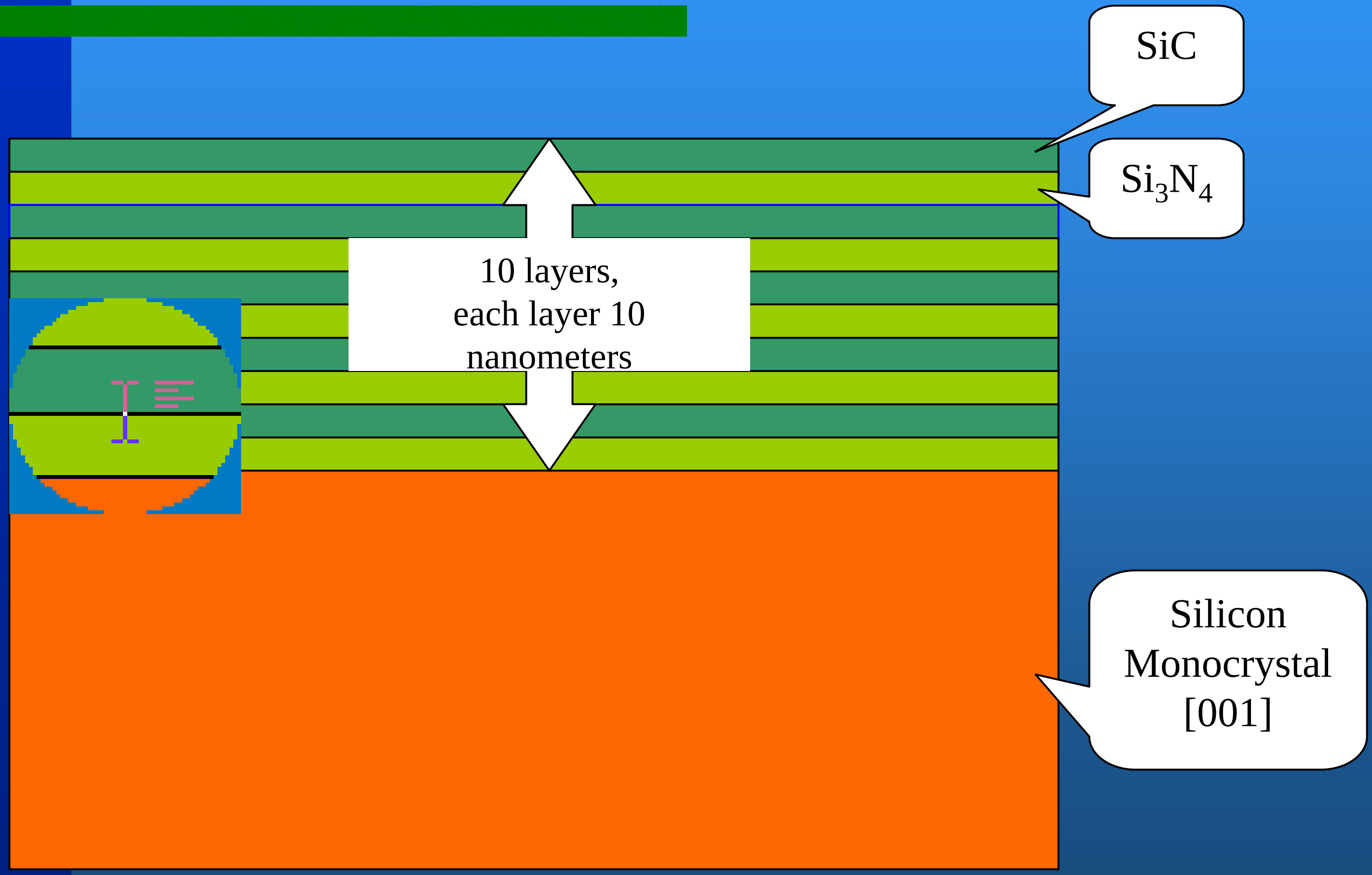


SH



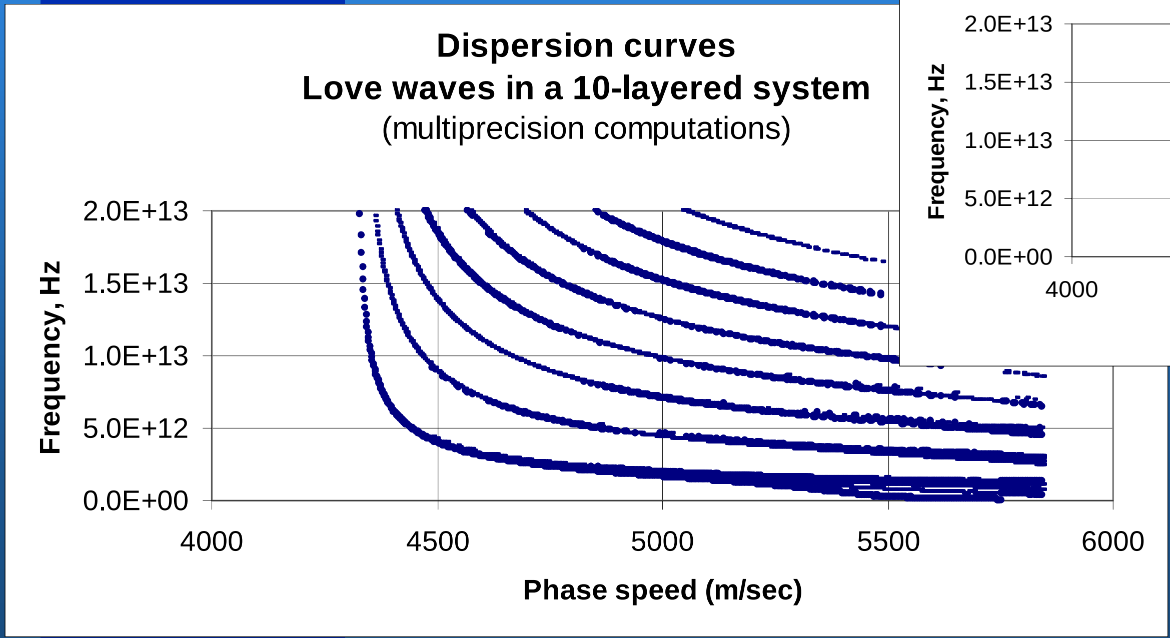


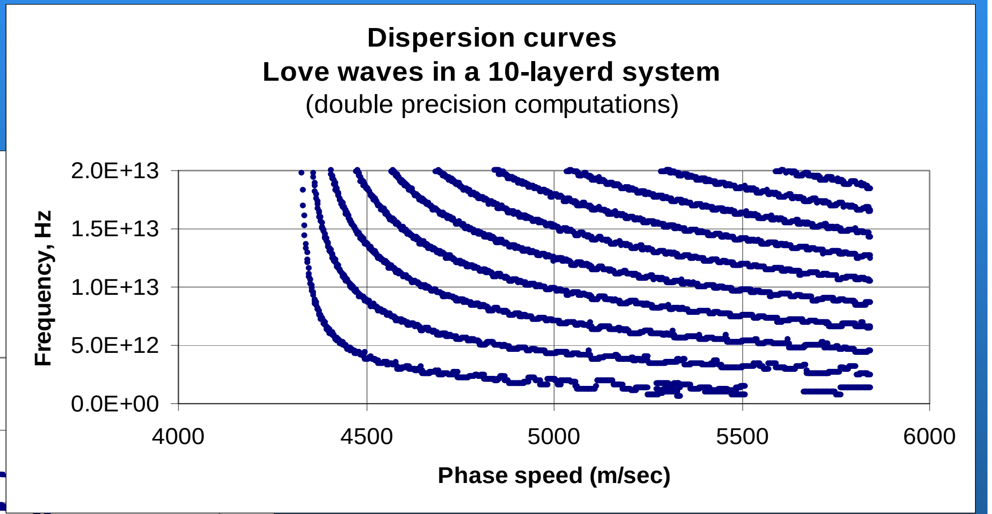
Love waves in multilayered medium



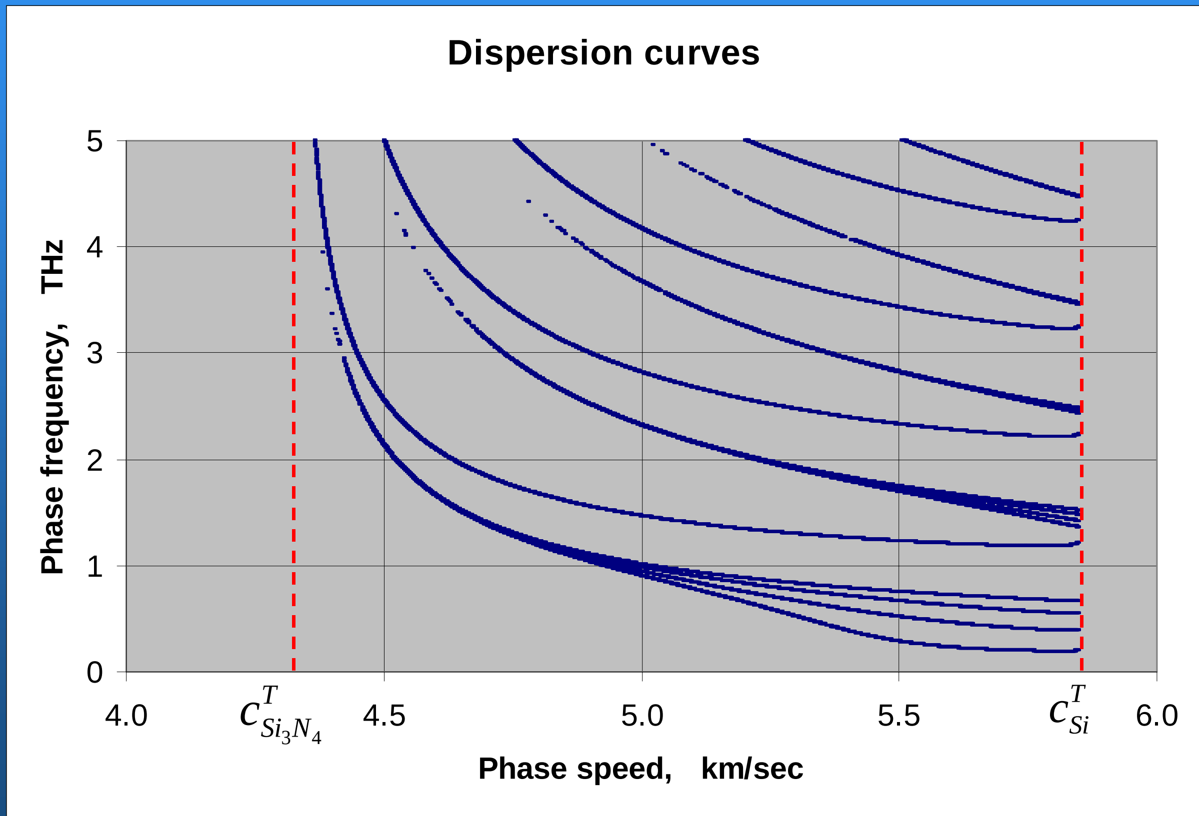
Numerical Example

Love waves in multilayered medium. Dispersion curves (10-layered plate)

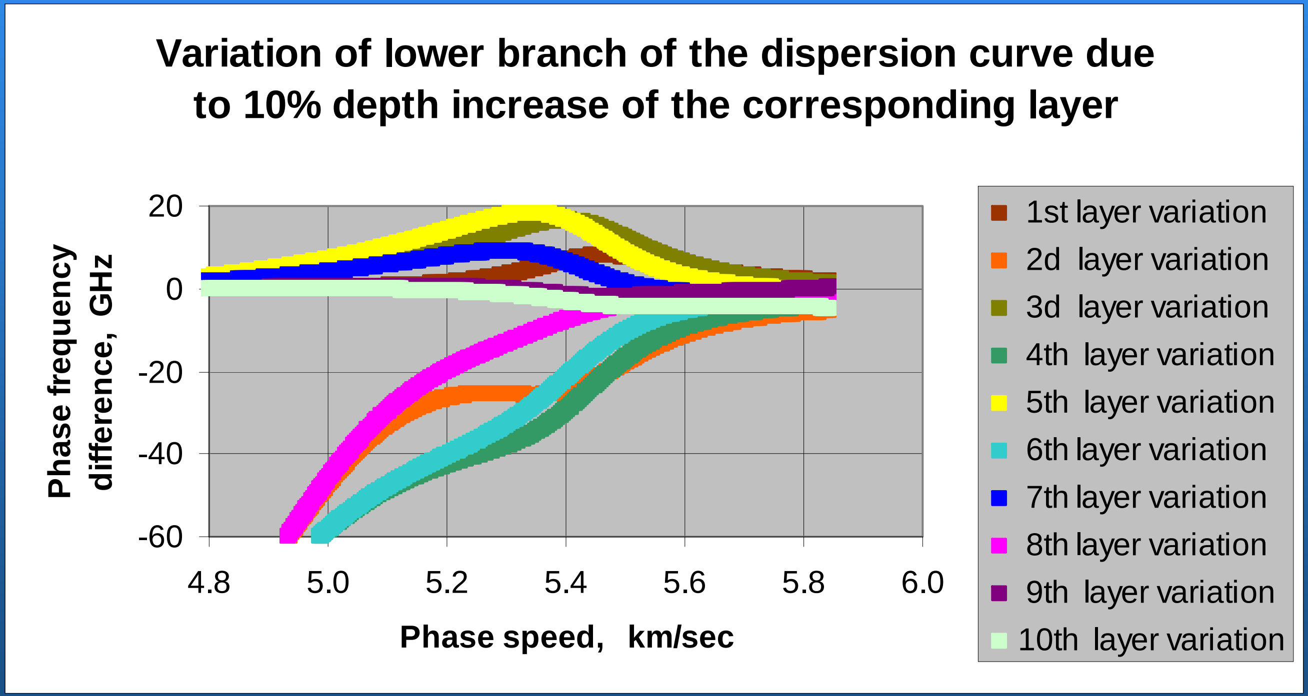




Love waves in multilayered medium. Dispersion curves (10-layered plate)



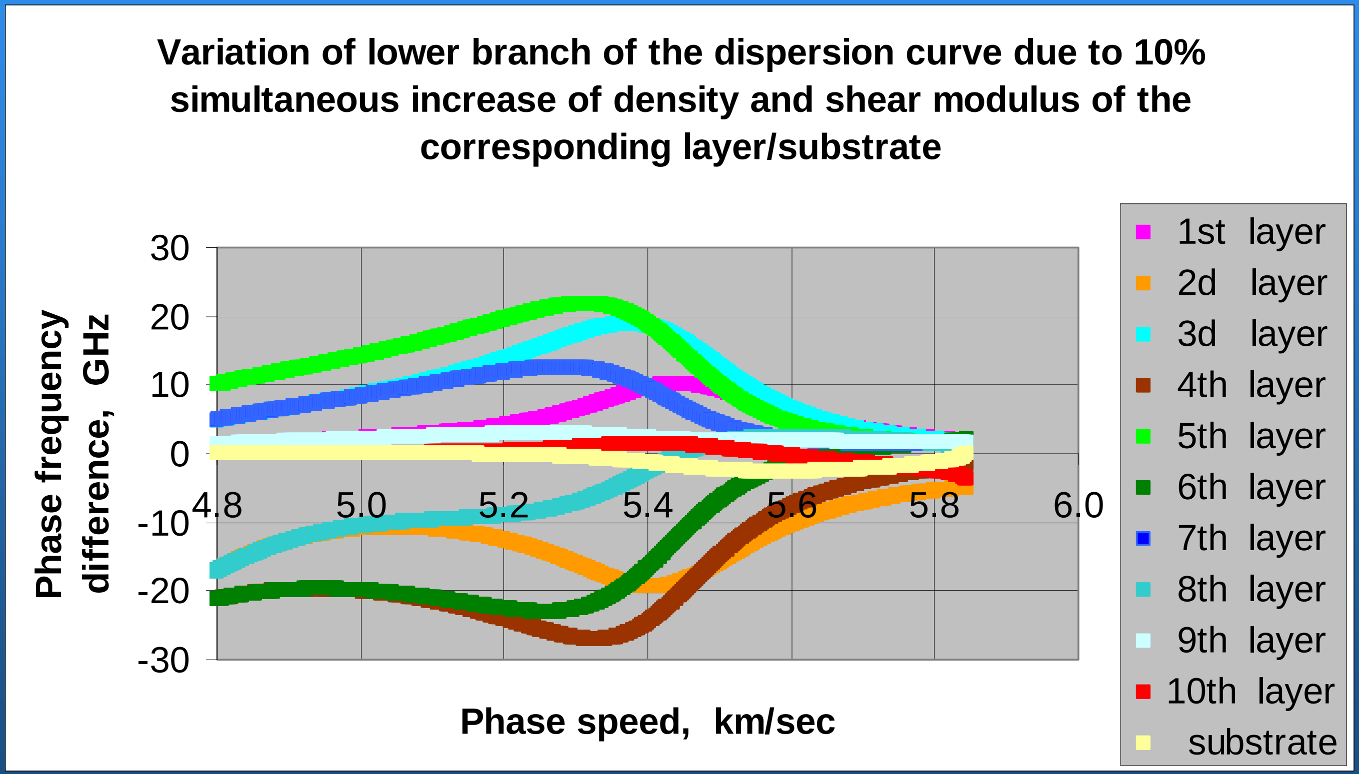
Love waves in multilayered medium. Layer thickness variation



Numerical Example

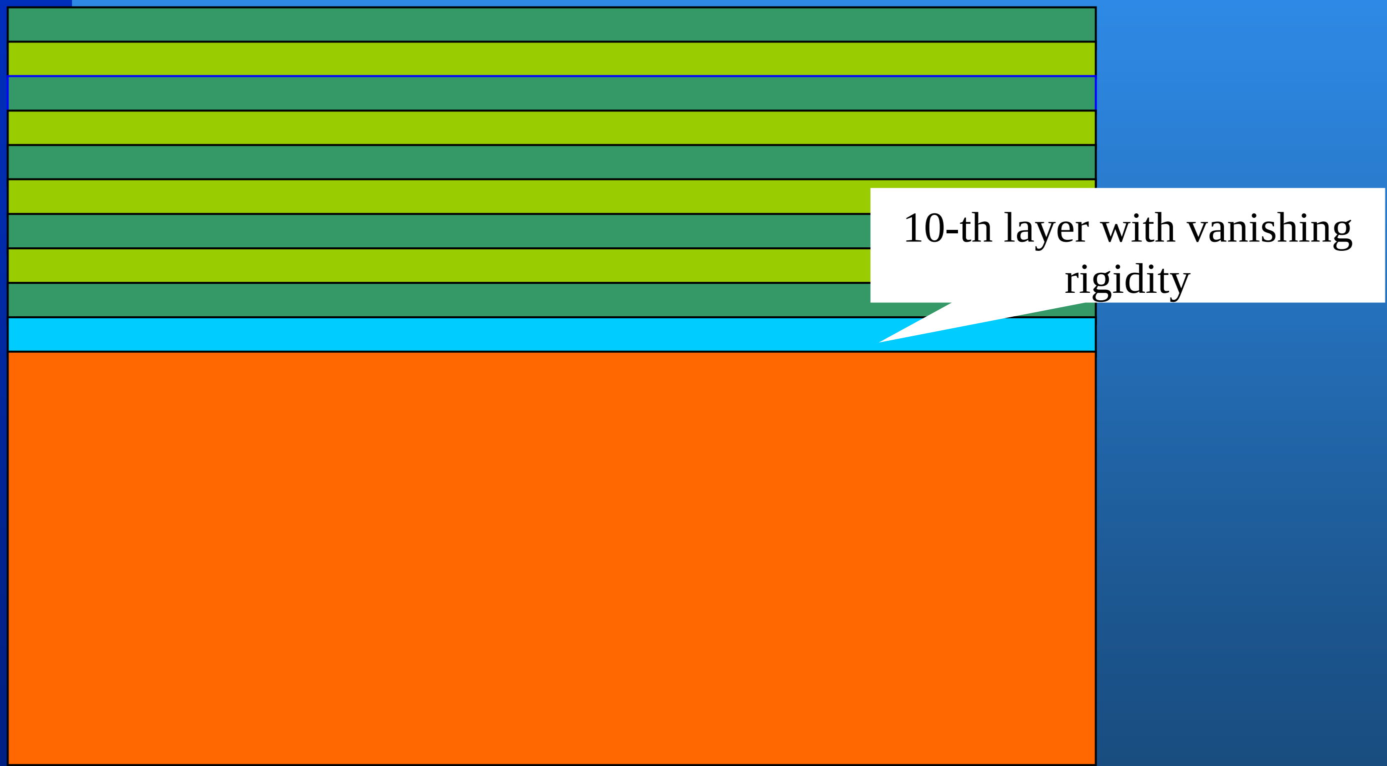
Love waves in multilayered medium.

Simultaneous variation of density and shear modulus of the corresponding layer/substrate



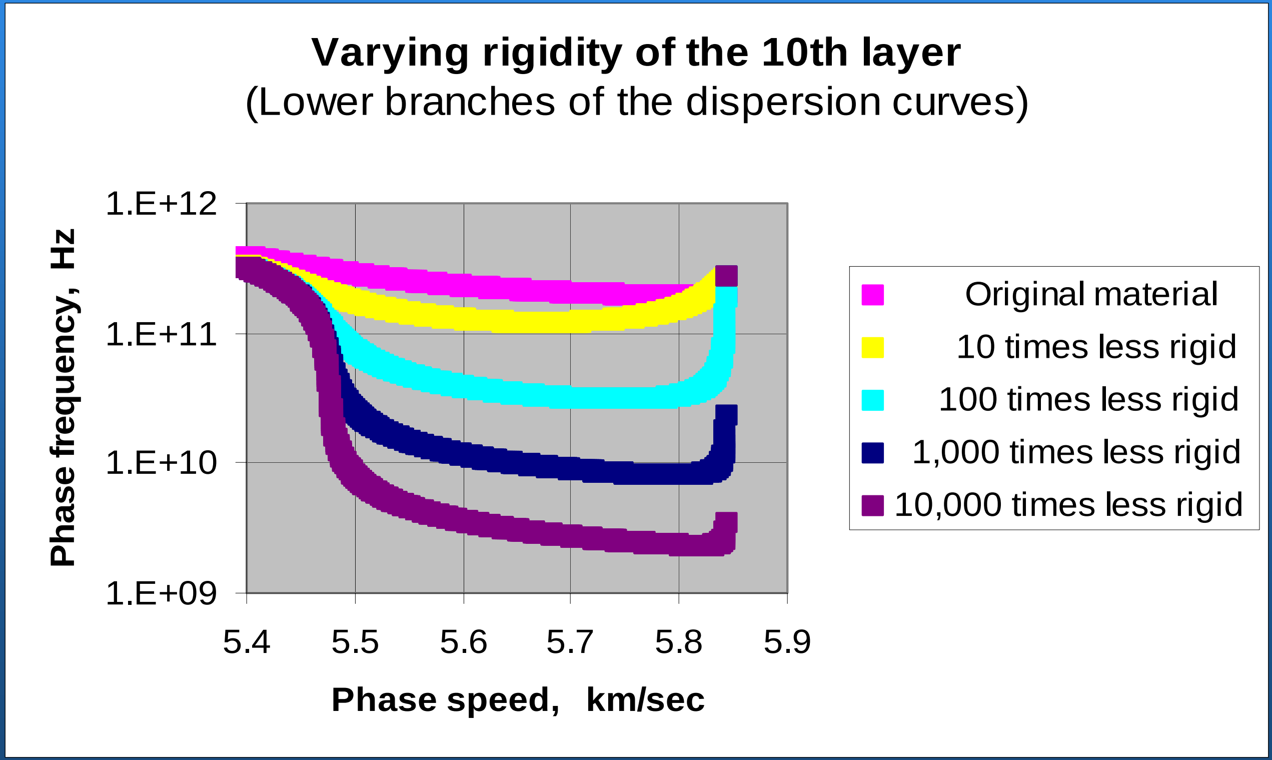
Love waves in multilayered medium.

Delamination of the 10-layered plate from the substrate



Love waves in multilayered medium.

Defoliation of the 10-layered plate from the substrate



Numerical analysis:
SH waves in stratified plates

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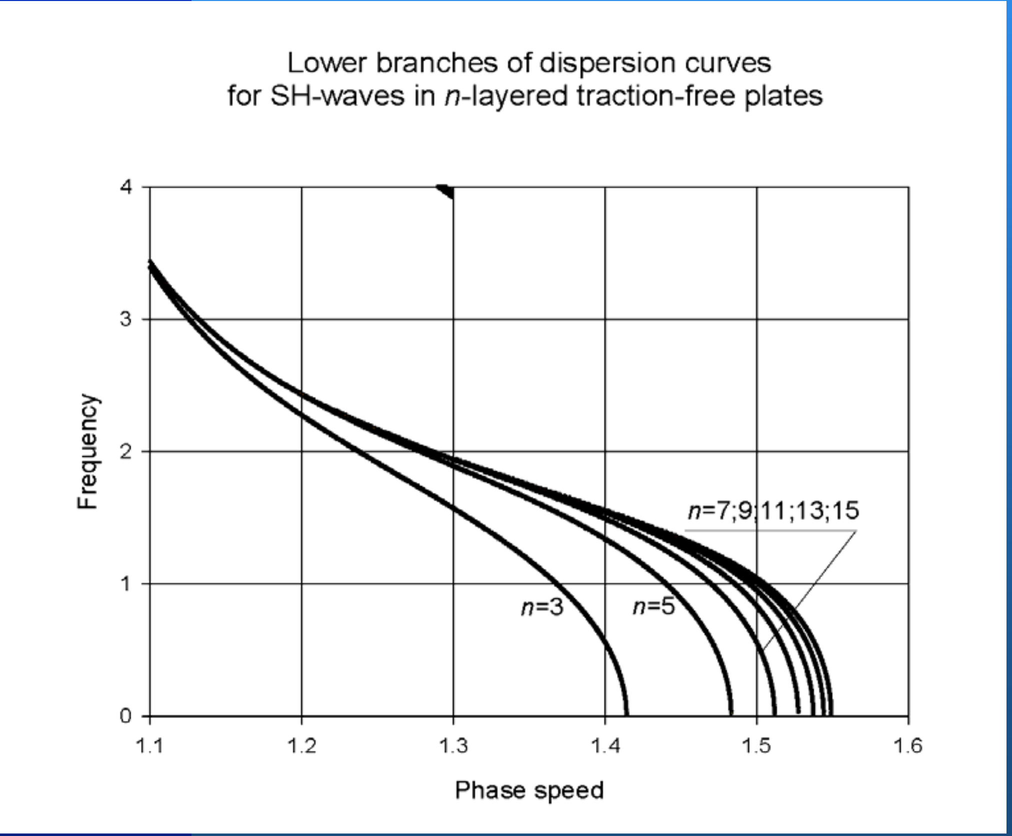
SH waves in multilayered plates

Lower mode dispersion curves for traction-free plates with different number of layers



Plates with alternating isotropic layers:

$$h_1 = \ldots = h_n = 1$$

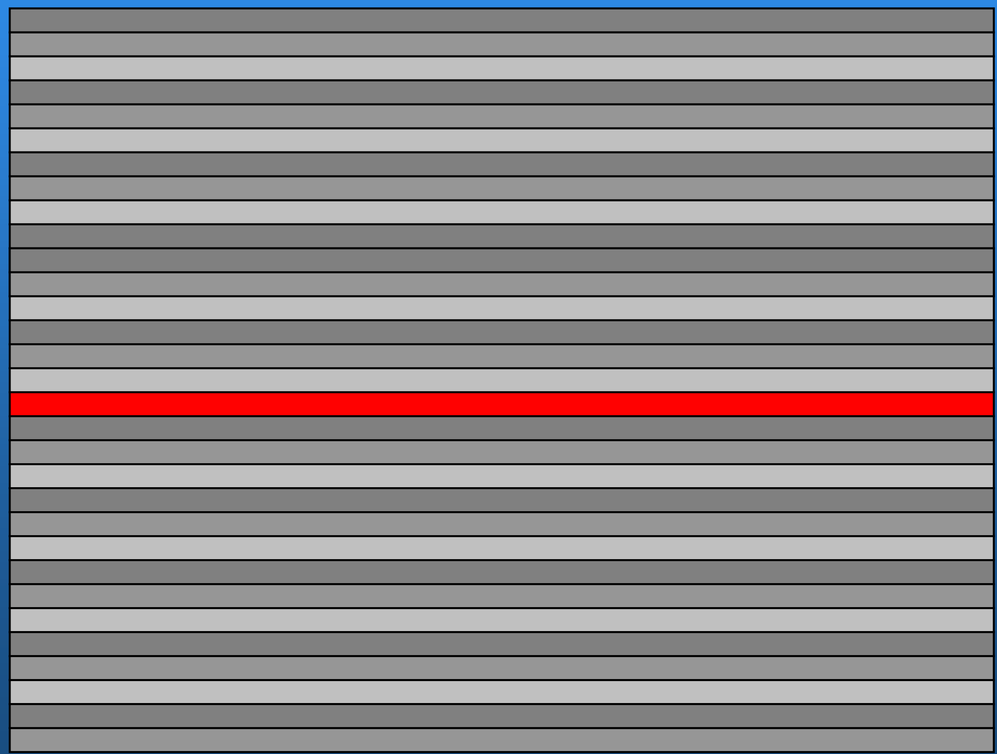
$$\rho_1 = \ldots = \rho_n = 1$$

$$\mu_1 = 1;\ \mu_2 = 4;$$

$$\mu_3 = 1;\ \ldots$$

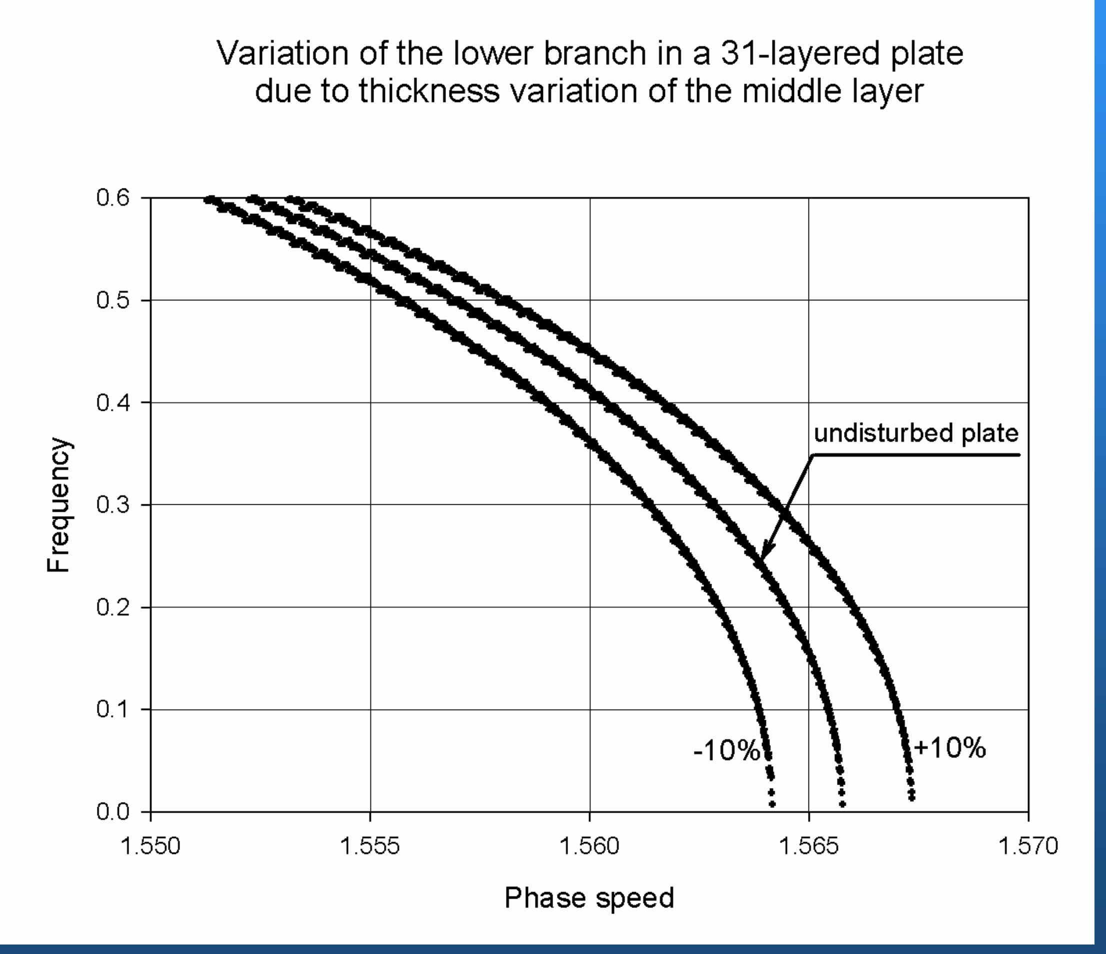
SH waves in multilayered plates

A 31-layered plate



SH waves in multilayered plates

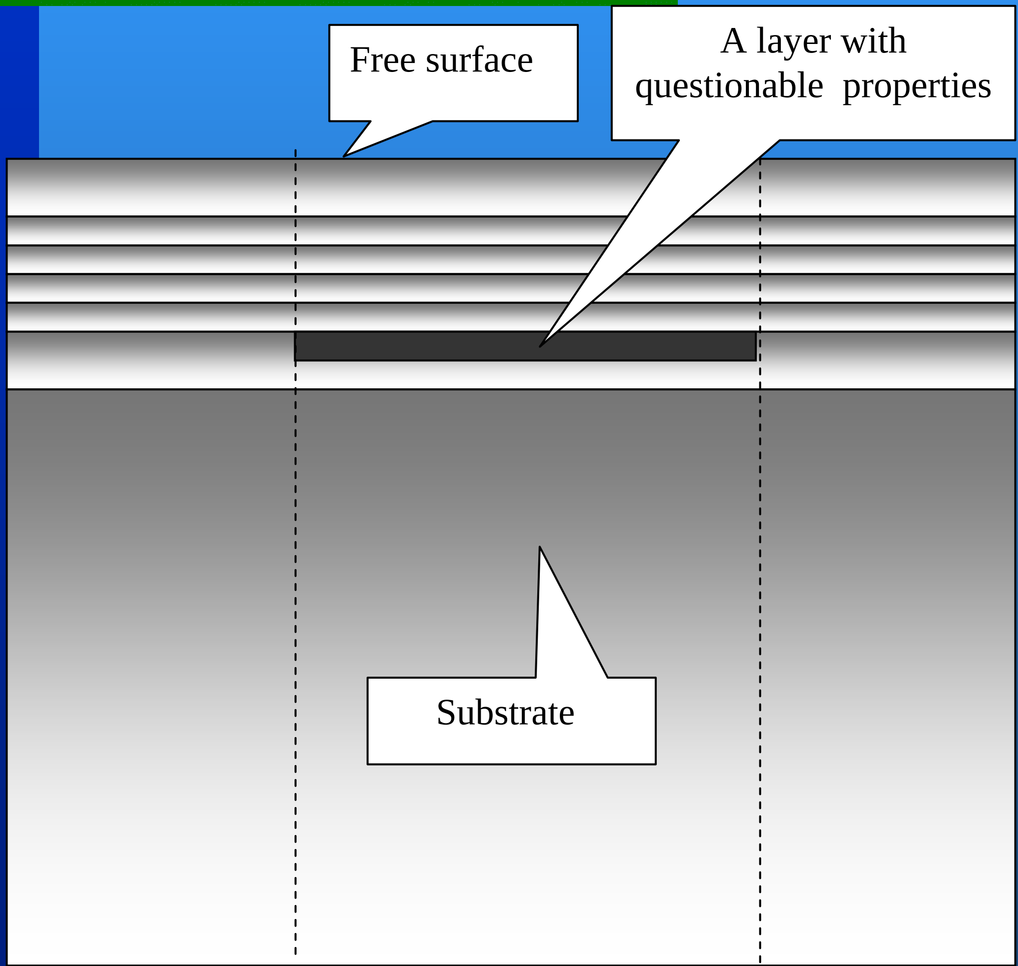
SH waves in multilayered plates



Possible applications of analyses for Love and SH waves propagating in stratified media

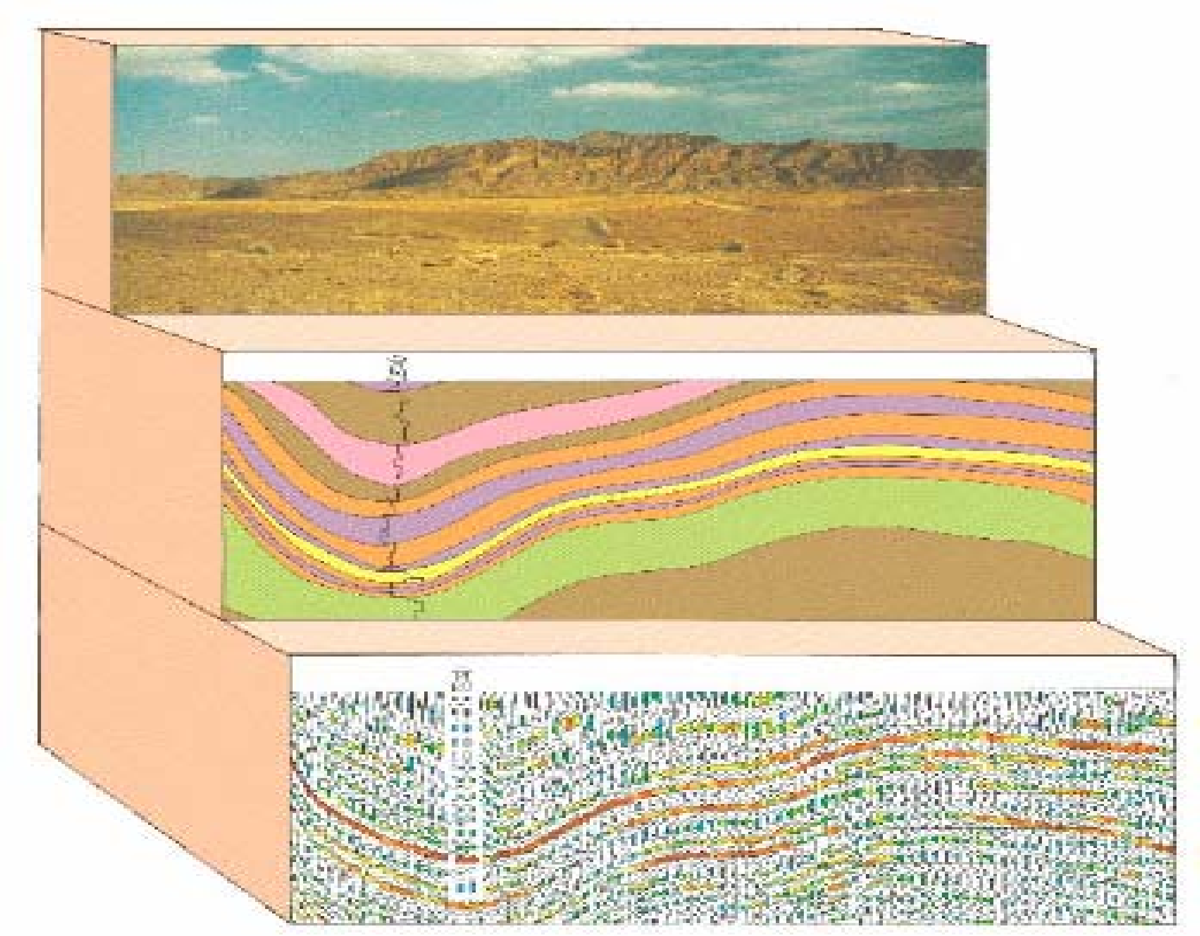
Possible applications

Determination of physical properties of the internal layer(s) with questionable properties by analyzing the dispersion relations for Love/SH waves



Possible applications

Application to geomechanics



Principle possibility to analyze depth, geometrical and physical properties of the questionable layers (water or oil saturated) by the dispersion curve analysis

Possible applications

Application to glue laminated timber structure analysis

Principle ability to analyze physical properties, presence of cracks, flaws, and delaminations by the dispersion curve analysis