Numerical modeling of damage

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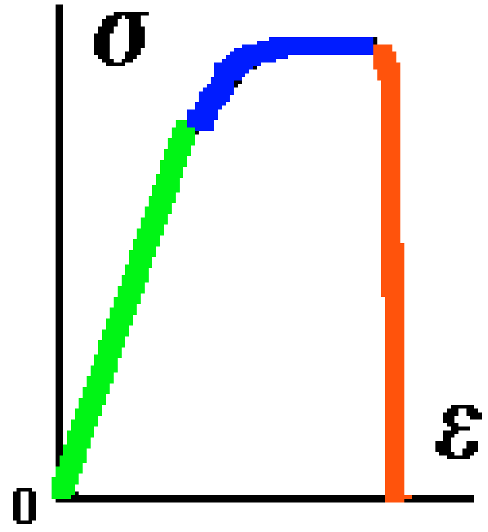
1. Elastic plastic flow theory

$$\partial_t \boldsymbol{\sigma} = E : (\partial_t \boldsymbol{\varepsilon} - \partial_t \boldsymbol{\varepsilon}_p) \qquad \partial_t \boldsymbol{\varepsilon} = (\partial_x \mathbf{u})_s$$



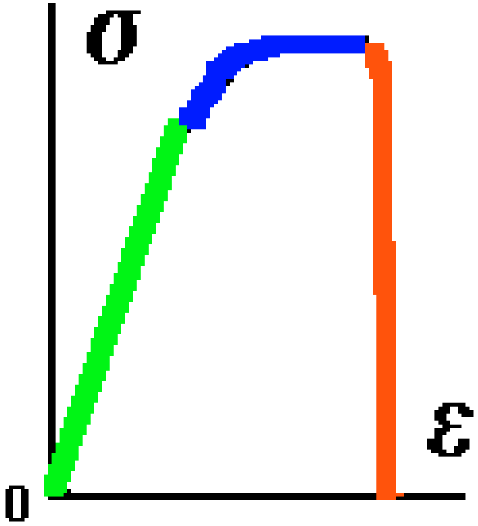
$$\partial_t \boldsymbol{\varepsilon}_p = H(\|\boldsymbol{\sigma}\| - \sigma_s(\|\boldsymbol{\varepsilon}\|))\lambda : \boldsymbol{\sigma}$$

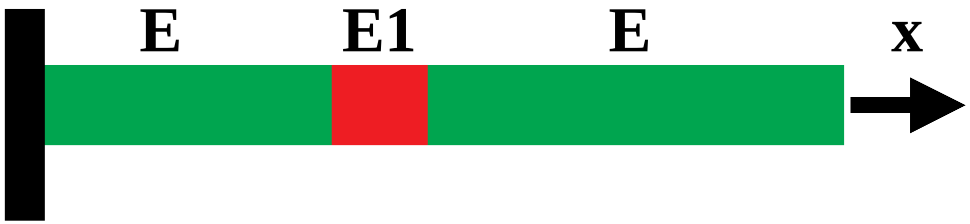
$$\begin{cases} \partial_t \boldsymbol{\sigma} = \mathbf{E}_{(ep)}(\boldsymbol{\varepsilon}, \boldsymbol{\varepsilon}_p) : \partial_x \mathbf{u} \\ \rho \partial_t \mathbf{u} = \partial_x \boldsymbol{\sigma} \end{cases}$$

is not applicable to softening because of violation Drucker material stability condition and Hadamard correctness criterion

$$\partial_t \sigma : \partial_t \varepsilon = (\mathbf{E}_{(ep)} : \partial_x \mathbf{u}) : \partial_x \mathbf{u} < 0$$

2. State of elastic body with "damaged" zone

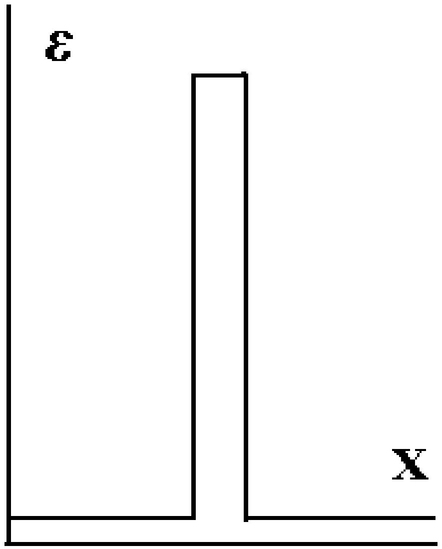




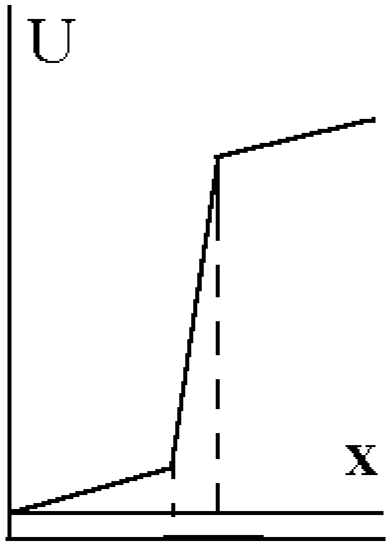
Green zones – virgine material (E)

Red zone - damaged material (E1)

Elastic moduli: E1 << E

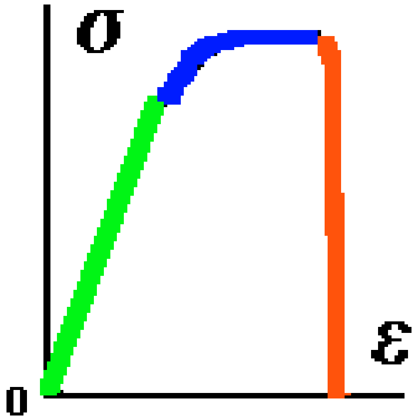


Deformation



Displacement

Damage may be simulated by degradation of elasticity



3. Damage theory (Kachanov, Rabotnov, 1958-59)

Damage and deformation are independent processes.

Damage criterion: $\Phi_\theta(\boldsymbol{\sigma}, \boldsymbol{\varepsilon}, \theta, \ldots, T, \ldots) \geq 0$

$$\partial_t \boldsymbol{\sigma} = \mathbf{E}_{(ep)} : \partial_x \mathbf{u} + (\partial_\theta \mathbf{E} \partial_t \theta) : (\mathbf{E}^{-1} : \boldsymbol{\sigma}) + \nabla \cdot (\nu_\sigma \nabla \boldsymbol{\sigma})$$

$$\rho \partial_t \mathbf{u} = \partial_x \boldsymbol{\sigma} + \nabla \cdot (\rho \nu_u \nabla \mathbf{u})$$

$$\partial_t \theta = H(\Phi_\theta) \lambda_\theta(\boldsymbol{\sigma}, \boldsymbol{\varepsilon}, \theta) + \lambda_\theta^{(0)}(x, t) + \nabla \cdot (\nu_\theta \nabla \theta)$$

Here Drucker material stability condition is violated

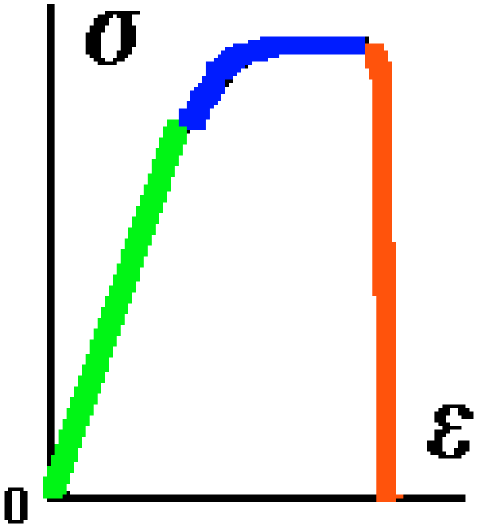
$$\partial_t \boldsymbol{\sigma} : \partial_t \boldsymbol{\varepsilon} = (\mathbf{E}_{(ep)} : \partial_x \mathbf{u}) : \partial_x \mathbf{u} + (\partial \boldsymbol{\sigma} / \partial \theta) \partial_t \theta : \partial_x \mathbf{u} \leq 0$$

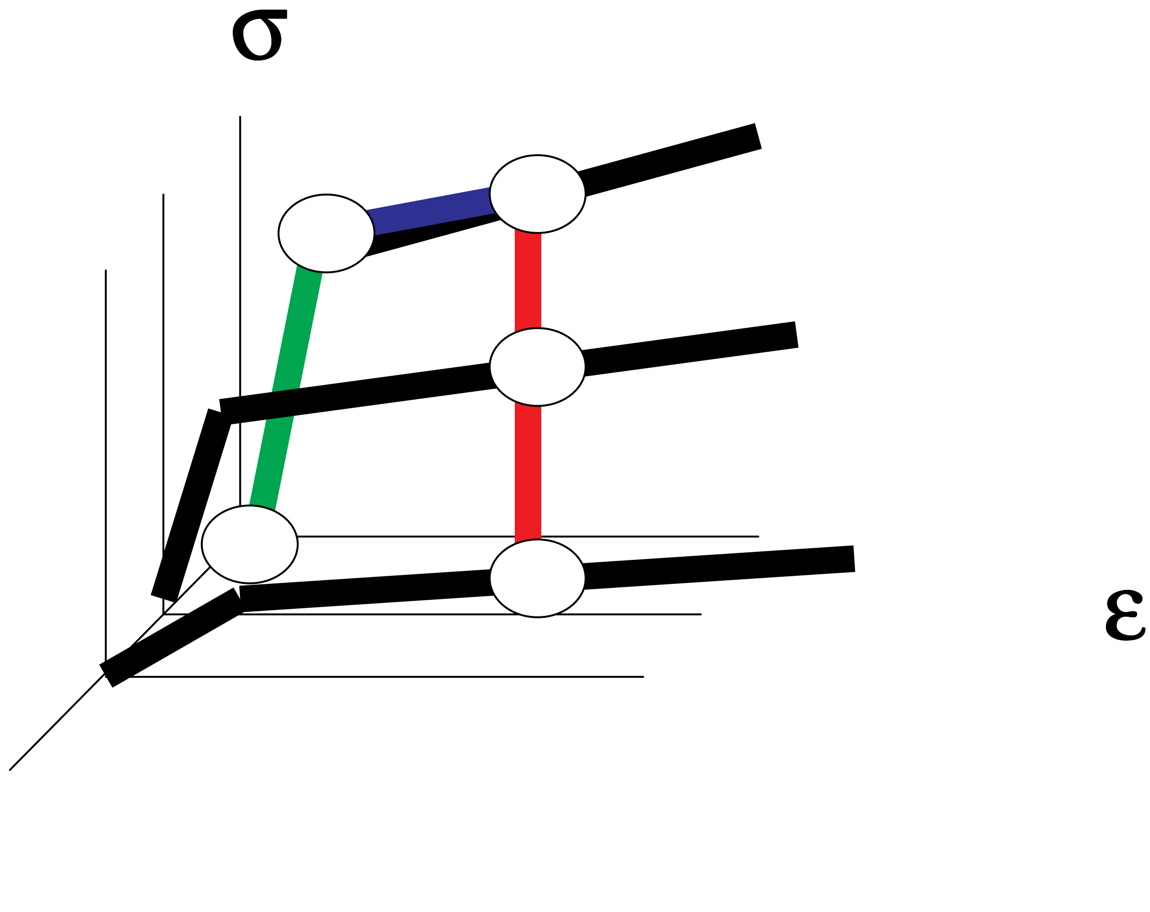
while Hadamard criterion is fulfilled, so the problem is correct

$$(\mathbf{E}_{(ep)} : \partial_x \mathbf{u}) : \partial_x \mathbf{u} > 0$$

Blue colour marks gradient theory regularization terms

4. Deformation diagrams in stress-strain-damage space





On stress-strain planes the deformation diagrams have no softening.
The softening takes place due to growing damage and degradating elasticity
The damage can grow due to non-thermomechanical actions.

6. NUMERICAL PROCEDURE

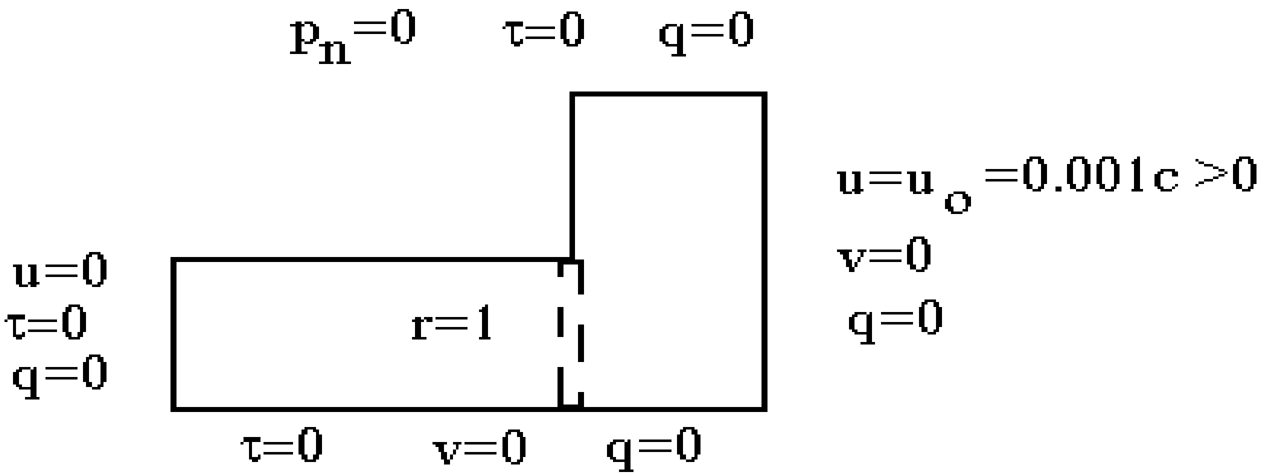
Implicit FE scheme (N.G.Bourago, V.N.Kukudzhanov, 1988)

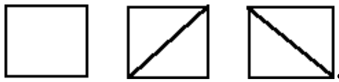
Basic features:

- Dynamic variational Galerkin formulation
- Unstructured Lagrangian meshes
- Implicit quasi-second order approximation in time
- linear and belinear approximations in space
- All discrete unknowns are nodal
- Diagonal mass matrix
- Quasi-Newtonian iterations
- Preconditioned matrix free conjugate gradient method
- «minimal» nonlinear monotonization
- Accuracy restriction for time step

More info can be found in: www.ipmnet.ru/~burago/papers

7. Damage of plane strain specimen under extension



Element shapes: , Spatial steps: 1/15, 1/30, 1/60.

Input data: $K_0 = 975$ $\mu_0 = 369$ $c_o = \dfrac{K_o + 4/3\mu_o}{\rho_o} = 1$ $\Phi_p = \sigma':\sigma' - 1$

$c_V = 1$ $k_T = 1$ $\beta = 0.0001$

$K = K_0 e^{-1000\theta}$ $\mu = \mu_0 e^{-1000\theta}$ $\partial_t \theta = H(\Phi_\theta)1000$ $\Phi_\theta = \varepsilon_{max} - 10^{-2}$

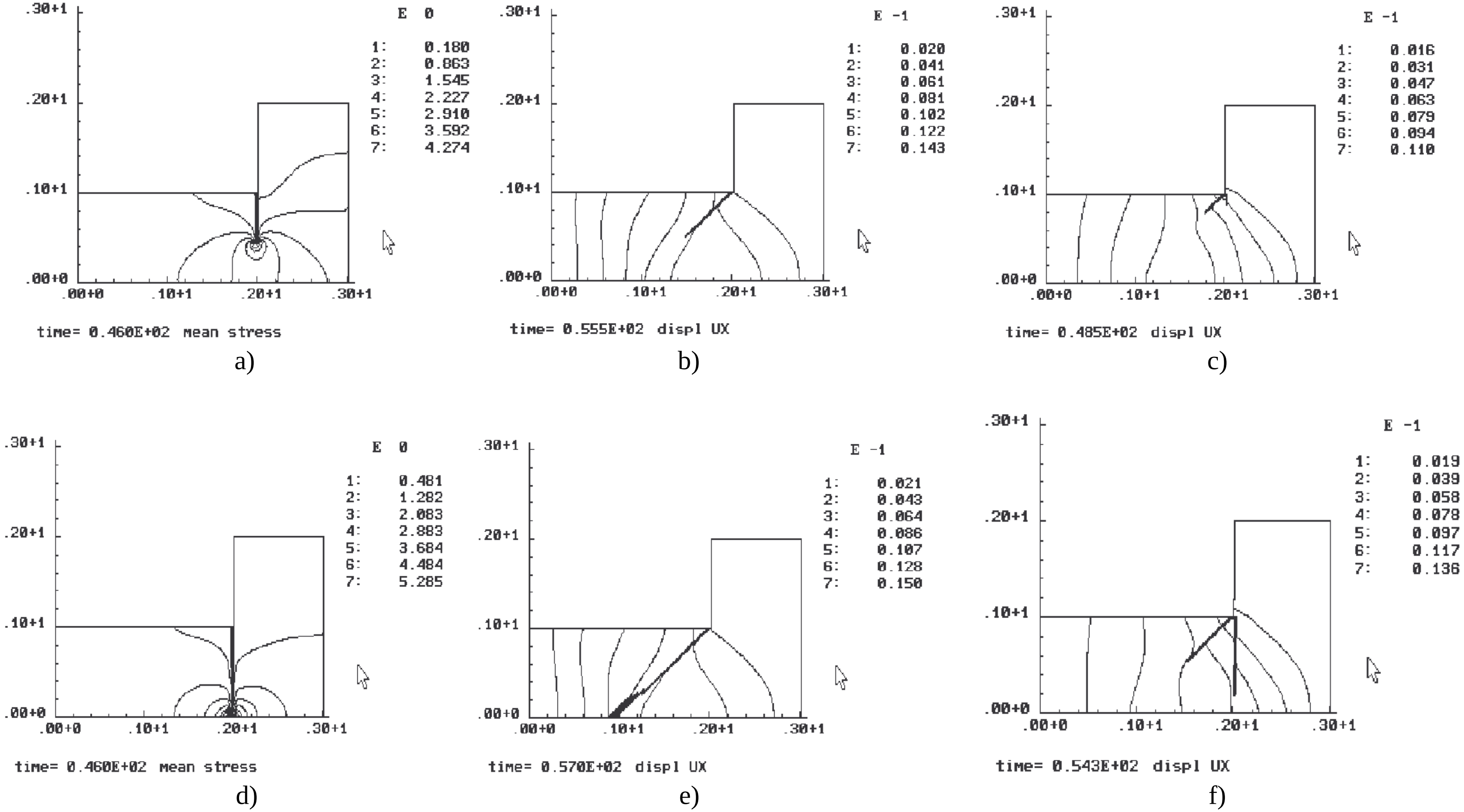
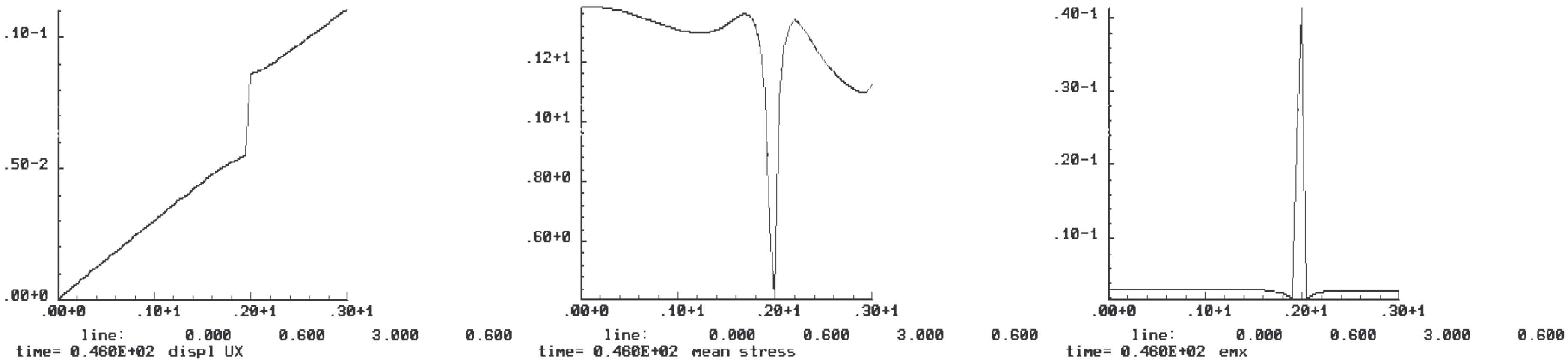
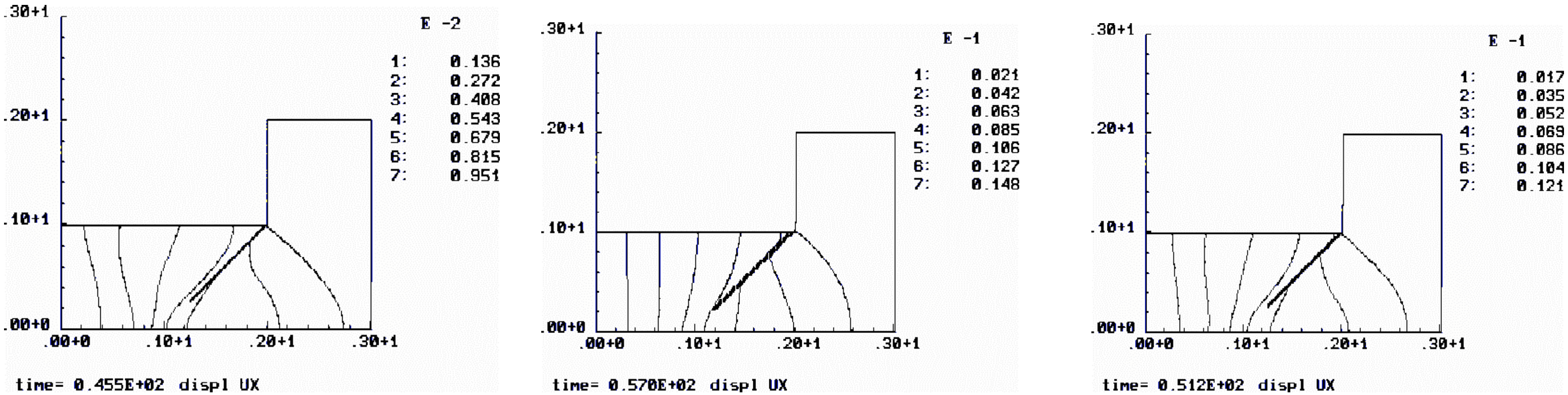


Fig. 2. Modes of damage for elastic material (a,d), elastic plastic material (b,e), elastic plastic material under heating (c,f).



a) b) c)

Fig. 3. Damaged state: typical graphs of horizontal displacement (a), mean stress (b) and maximal principal strain (c) along horizontal line (0,0.6,3,0.6)

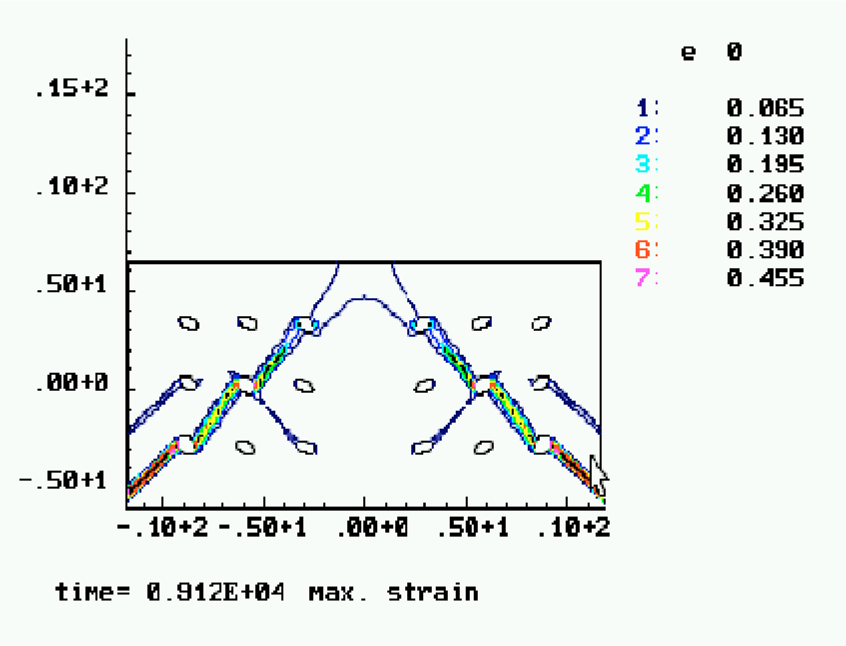


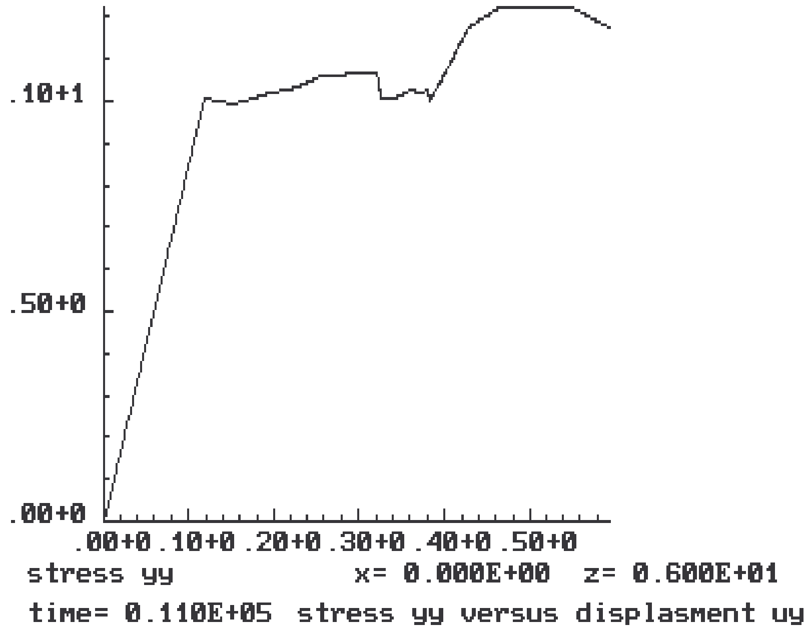
a) b) c)

Fig. 4. "Macrocrack" and contur lines of horizontal displacement illustrate the convergence of numerical solutions. Graphs correspond to various meshes with spatial steps (1/15 (a), 1/30 (b), 1/60 (c)).

Damage of specimen with macrodefects (pores) and microdefects

- micropores (volumetric plasticity)
- dislocations (deviatoric plasticity),
- microcracks (damage parameter)





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- isotropic elasticity, associated flow rule and Garson's plasticity, damage criterion in terms of maximal principal strain, constant damage growth rate, almost instant degradation of elasticity..

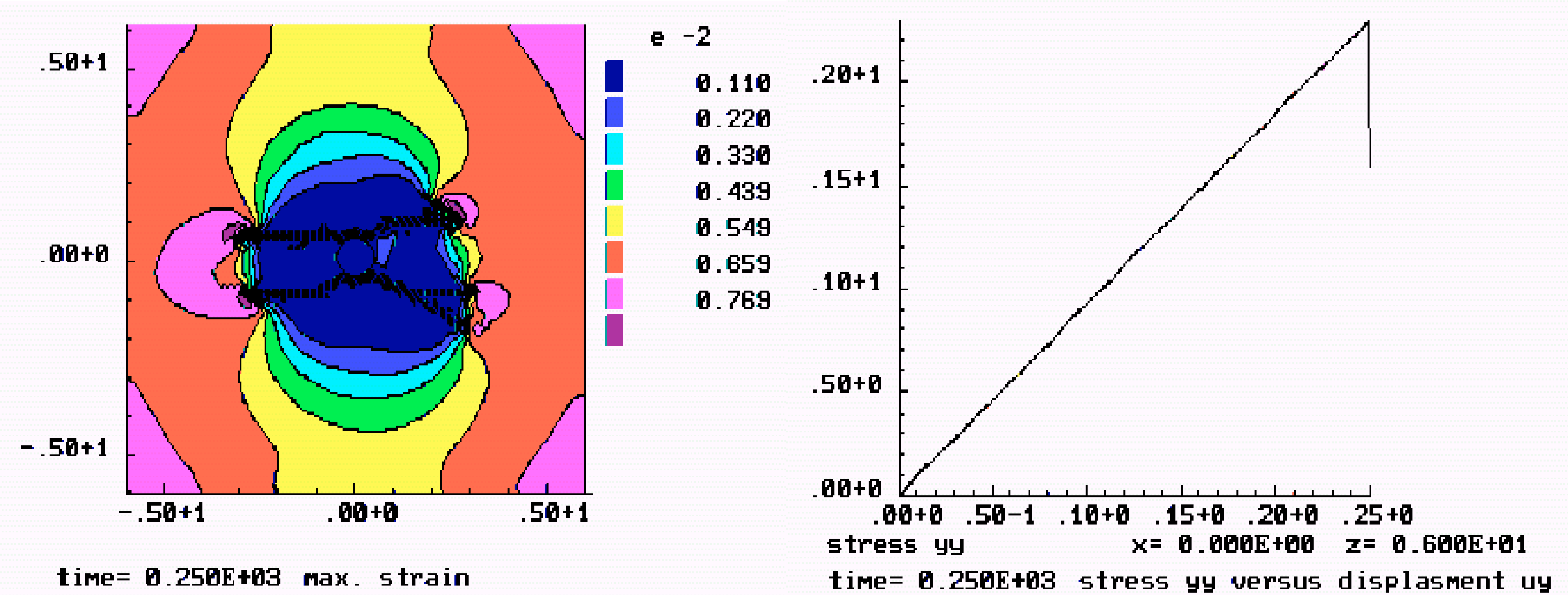


Fig. 5. Damage of stretched plate with rigid curcular inclusion. Black narrow zones are developed macrocracks. Colour zones indicate the value of deformation $\varepsilon_x + \varepsilon_y$. The graph on the right draws the dependence of the stress σ_y on the displacement U_y in the point ($x = 0$, $y = 6$).

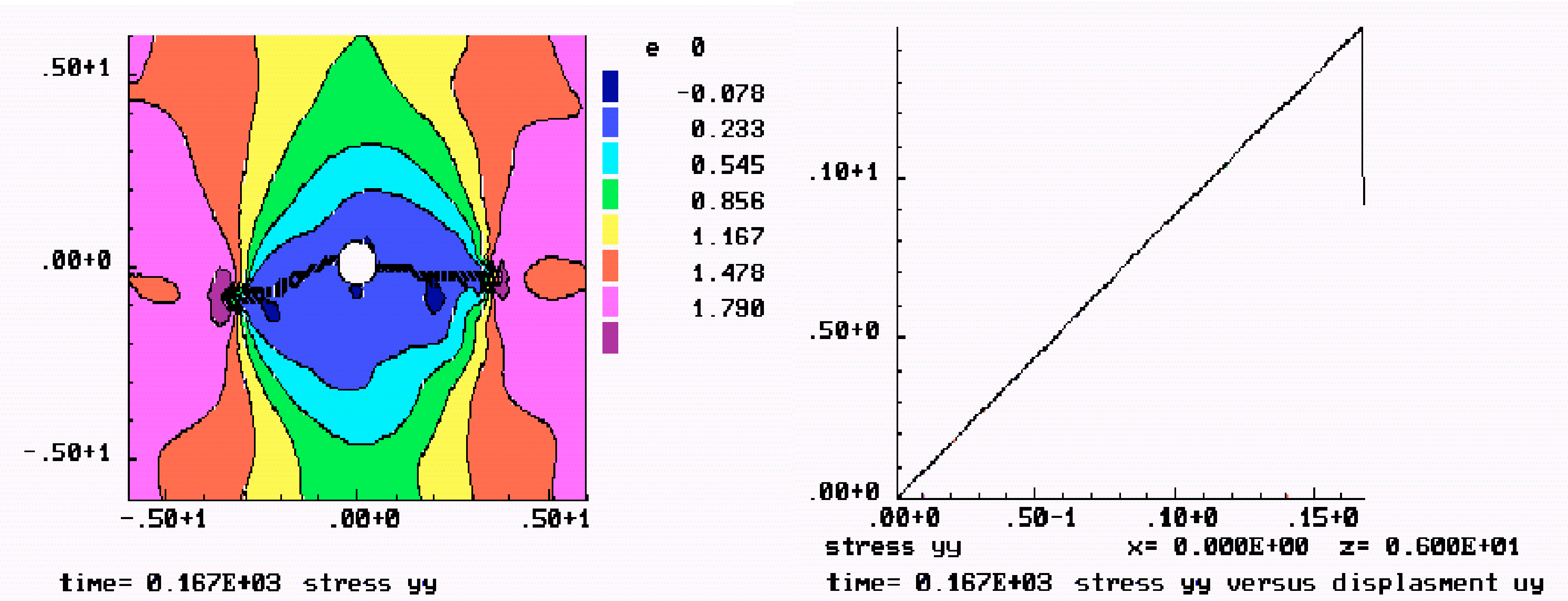


Fig. 6. The damage in stretched plate with circular pore. Black narrow zones indicate developing macrocracks. Colour zones indicate the value of deformation $\varepsilon_x + \varepsilon_y$. The graph on the right indicates the dependence of the stress σ_y on the displacement U_y in the point ($x = 0$, $y = 6$).

In both cases of Fig. 1 and 2 the matrix material is elastic.

Explanation: The difference in the character of damage processes is explained by the different character of strain concentration near the pore and near the rigid inclusion. This may be seen in Fig. 7.

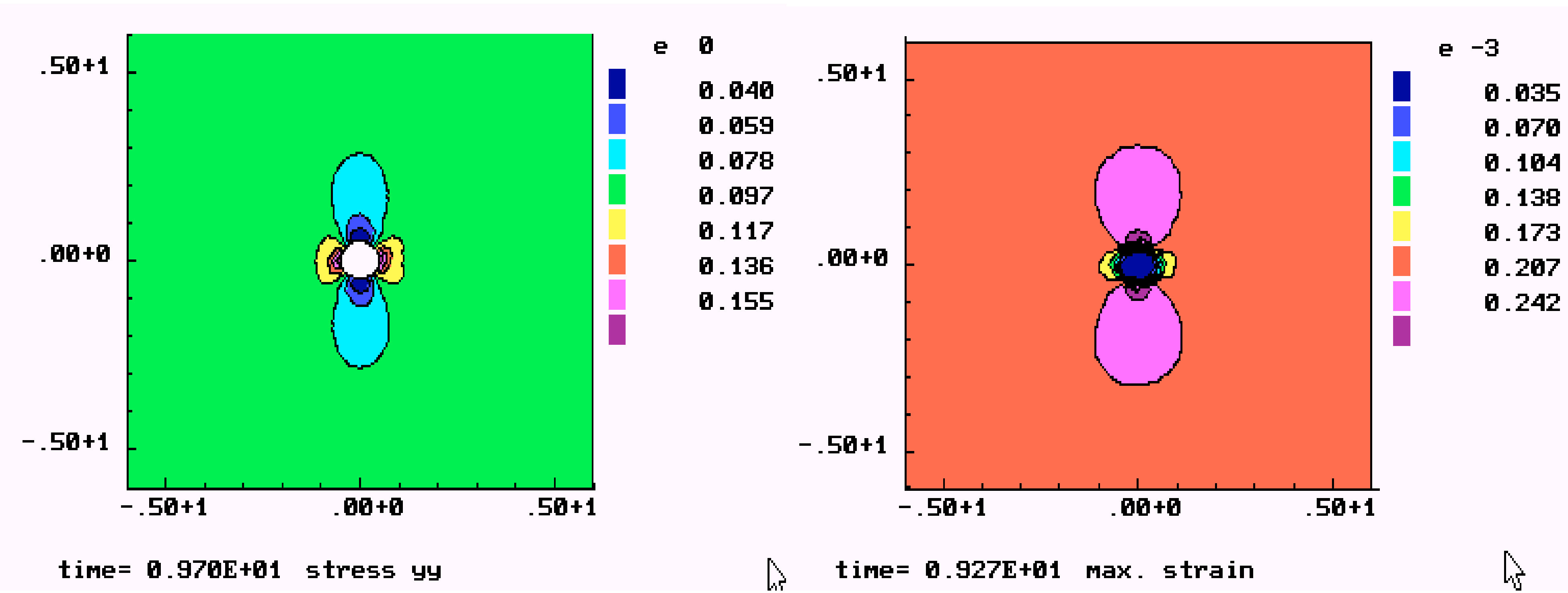


Fig. 7. The picture of strain concentration in vicinity of the pore (on the left) and near rigid inclusion (on the right). The dark violet color corresponds to the maximal deformation.

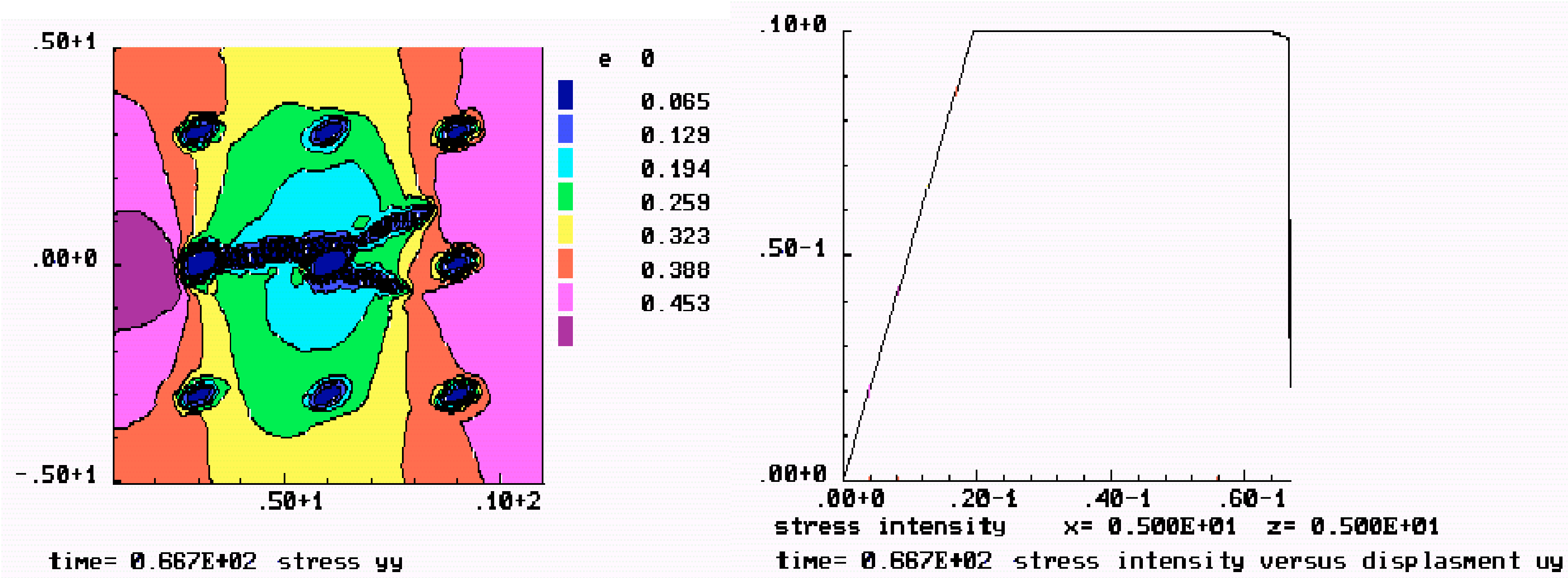


Fig. 8. The damage in stretched plate with rigid elliptic inclusions, rotated through the angle 30^{o}. The damaged zones, the distribution of stresses σ_{yy} (on the left picture) and calculated deformation diagram (on the right picture) are presented. The mesh contained 2100 elements.

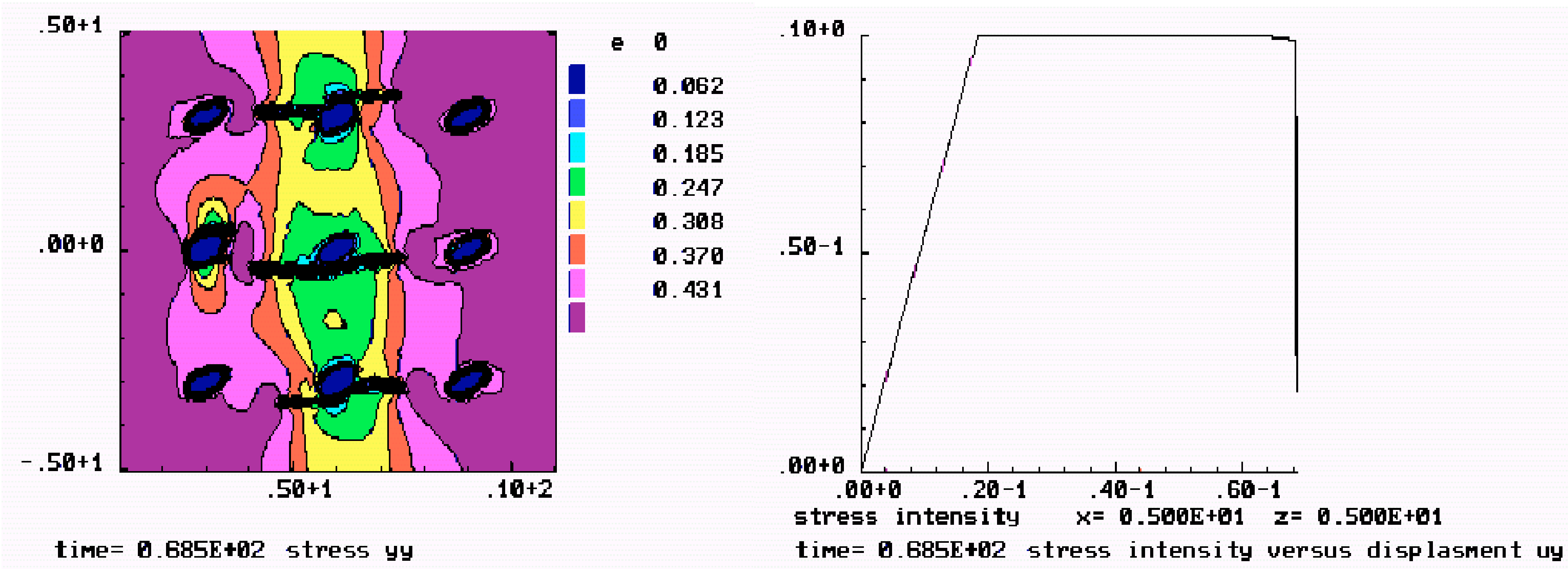
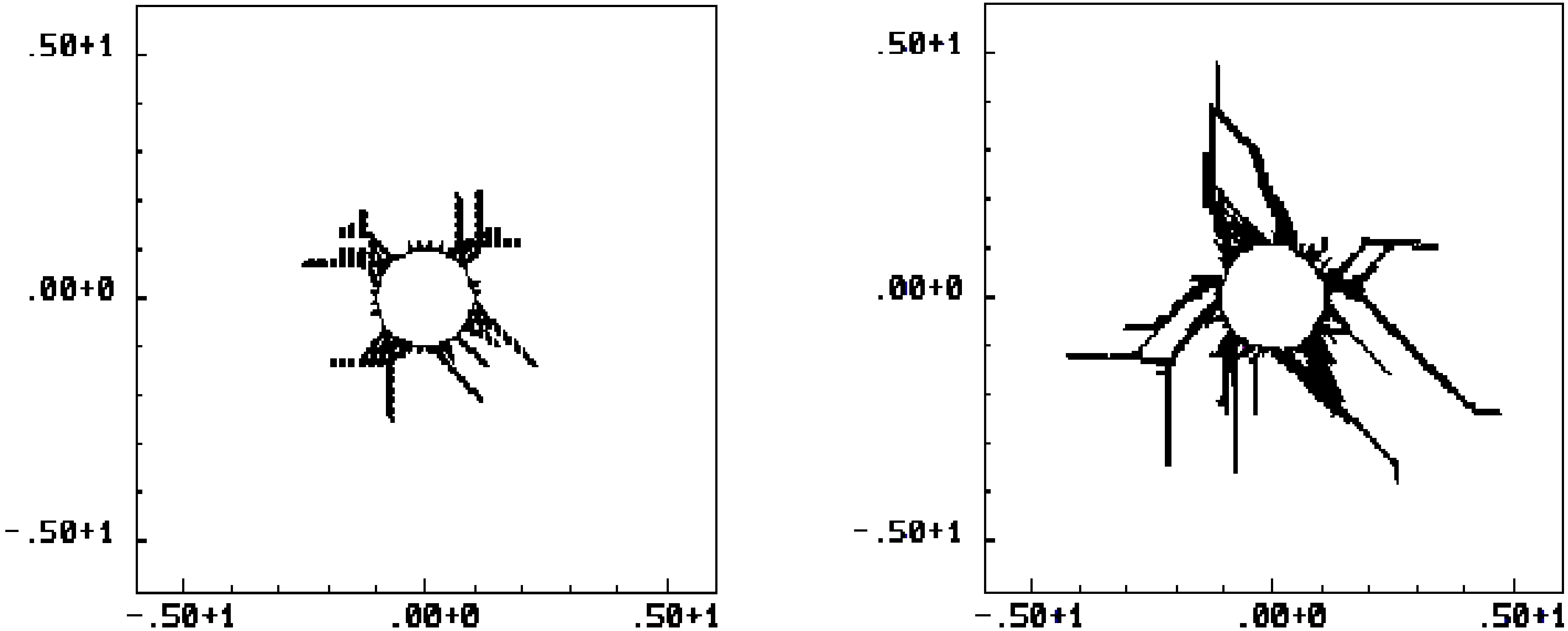


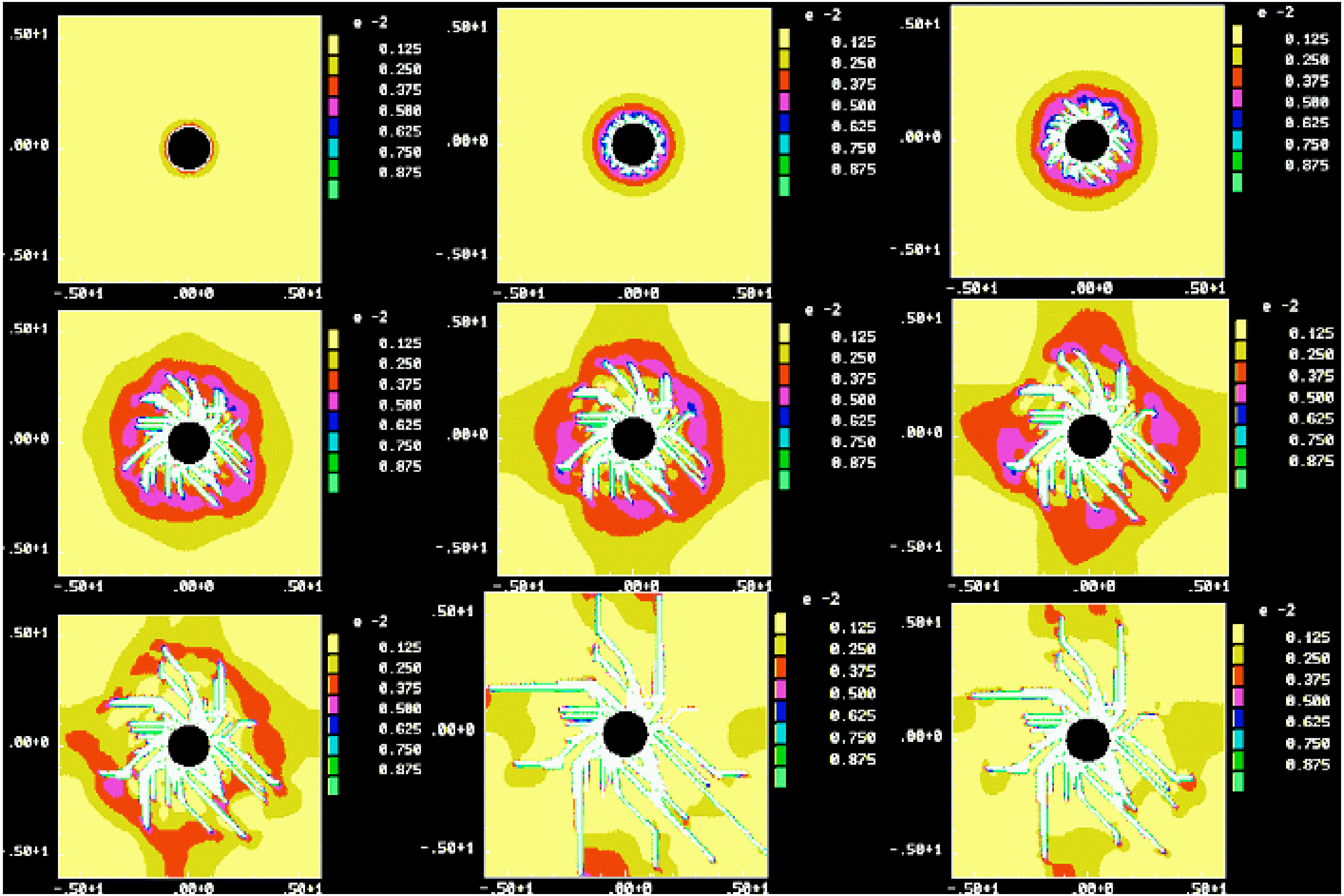
Fig. 9. The damage in stretched plate with rigid elliptic inclusions, rotated through the angle 30^{o}. The damaged zones, the distribution of stresses σ_{yy} (on the left picture) and calculated deformation diagram (on the right picture) are presented. The mesh contained 8400 elements.

Deformation diagrams (integral characteristic of the process) are practically coinsided, while the details of damage localization are different (local pecularities of the damage process).

The damage in stochastically inhomogeneous rocks near the drillhole under action of internal pressure



Maximal deflection of strain limit in damage criterion 5% (left) и 20% (right).



The development of macrocracks in rocks near drillhole

7. CONCLUSION AND PERSPECTIVES

The following theoretical model features are most important:

- Damage criterion should be formulated in terms of strain concentration coeffitients (because deformations are not corrected in most of theories in contrast to stresses and materal parameters).
- Almost instant degradation of elasticity with damage growth is required for localization of damage as contact discontinuities or macrocracks.
- Minimal nonlinear monotonization near macrocracks is required in order to prevent spurious short wave oscillations of numerical solutions.
- Resistance of damaged material in respect to compression is necessary in order to provide the positiveness of elementary volumes.
- Taking into account the inertia forces is required to support the correctness of boundary value problems if the fragmentation takes place.
- Accuracy control by means of restriction of strain increments is needed because of wide range of possible damage process rates regardless of the loading rate.

Questions for future research

Comparison of various theoretical approaches to damage simulation

Damage theories

Gradient theories

Elasto-visco-plastic models

Choice of damage parameter

Formulation of damage criterion

Role of damage kinetics

Dependence of elasticity on damage

Error estimate studies

Stability and convergence of numerical solutions are still hot problems.

Optimization of numerical models

3-D damage modelling

ADJUSTMENT OF THEORY AND EXPERIMENT

THE END