Proc. of 5th USSR Symposium, Alma-Ata, 1973, P. 101-107.

НАУЧНЫЙ СОВЕТ ПО ПРОБЛЕМЕ ПРОЧНОСТИ И ПЛАСТИЧНОСТИ АН СССР ИНСТИТУТ
ПРОБЛЕМ МЕХАНИКИ АН СССР ИНСТИТУТ МАТЕМАТИКИ И МЕХАНИКИ АН КазССР
МОСКОВСКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ им. М. В. ЛОМОНОСОВА

РАСПРОСТРАНЕНИЕ УПРУГИХ И УПРУГОПЛАСТИЧЕСКИХ ВОЛН

(Материалы V Всесоюзного симпозиума)

Издательство «НАУКА» Казахской ССР АЛМА-АТА-1973

Proc. of 5th USSR Symposium, Alma-Ata, 1973, P. 101-107.

Propagation of elastic-visco-plastic waves in mediums with delay of plasticity

BURAGO N.G., KUKUDZHANOV V.N.
(Moscow)

The propagation of waves in rods is considered taking into account the delay of plasticity property. It is assumed that alastic-visco-plastic material behavior is governed by the Cotrell's delay of plasticity condition. Simple numerical method for solving such problems is proposed. It is shown that this method allows obtaining solutions for the Rabotnov's [I] plasticity theory. Our numerical results are compared with analytical solution for this theory, obtained in [2].

§ 1. Many experimental studies are devoted to the phenomenon of delay of plasticity and relate mostly to quasi-dynamics range of loading rate [3]. There are several theoretical studies [4—5], which contain definite models for description of delay of plasticity property for propagation of waves.

On the basis of dislocations theory and experiments Yu. N. Rabotnov proposed a theory of elastic plastic medium with delay of plasticity [1]. According to this theory the transfer of material from elastic to plastic state happens instantly under Cotrell's plasticity condition. This leads to appearance of strong shock waves which interact with elastic and [plastic waves and provide rather complicated picture of wave motion. Analytical solution can be constructed only for the case of simplest boundary conditions and idealized Prandtl strain-stress diagram [2].

To obtain solutions fo real strain-stress diagrams it need to use numerical methods. Multiple various shock waves in real problems make impossible exact consideration of all shocks. Therefore it needs to introduce some modified rheological relations in order to describe the transfer from elastic to plastic state. This leads to the becessity of definite smoothing of shocks and to possibility of through calculations.

In general, if real transfer is almost instant then it is possible not to care about its adequate description. It is enough that introduced additional terms of equations were suffucuently small and led to required smoothing of shocks (for instance, introduction of artificial viscosity in case of ideal gas).

During plastic deformation of solids the viscous fluxes affect significantly onto propagation of waves and the layer of transfer from elastic to plastic state has much more thickness than in gases. Therefore it is possible in spite of poor experimental data to introduce such rheological modificated models which reflect qualitatively material behavior in plastic state and simultaneously contain some freedom for quatitative description of experimental data. For materials with neglidgable dissipation such freedom may be used for simplification of solution process. This allows to use one and the same numerical method and besides one and the same computer code in both cases.

The Cotrell's plasticity condition read:

$$\int_0^1 \varphi(\sigma, T)dt \geq t_1 \qquad (1)$$

Assume that

$$\varphi(\sigma, T) = \left(\frac{|\sigma|}{\sigma_s} - 1\right)^n H\left(\frac{|\sigma|}{\sigma_s} - 1\right)$$

where σ_s is the static yield limit; t_1 и n are experimental constants, which are dependent on temperature *T*; t is time; σ is stress; $H(z)$ is Heavyside's function.

Following Yu. N. Rabotnov [I], assume that while the condition (1) is not fulfilled, the material ibehavior is described by Hooke's law, but if the condition is fulfilled then material transfers into plastic state. Assume that plastic state is described bt the Sokolovsky-Malvern equations accounting the influence of strain rate on lower yield limit and some other special fwatures of elastic plastic wave propagation [б]. Equation of state can be written in unified form:

$$\frac{\partial \varepsilon}{\partial t} = \frac{1}{E}\frac{\partial \sigma}{\partial t} + \frac{1}{\tau} H(\Theta)H(\kappa)F(\kappa)\text{sign}\sigma \qquad (2)$$

$$\Theta = \int_0^1 \varphi(\sigma, T)dt - t_1, \ \kappa = |\sigma| - f(|\varepsilon|)$$

Here the dependence $\sigma = f(\varepsilon)$ relate to static strain-stress diagram of uniaxial loading, $F(z)$ is a function of viscosity, E is Young's module, τ is viscosity constant measured in seconds.

Consider flat waves propagation in a rod of material subjected to equation of state (2). Adding to Eq. (2) the momentum equation and relation between strain and velocity

$$\frac{\partial\sigma}{\partial x} = \rho\frac{\partial v}{\partial t}, \qquad \frac{\partial\varepsilon}{\partial t} = \frac{\partial v}{\partial x} \tag{3}$$

We get closed system of equations for description of medium motion. Introduce dimensionless variables:

$$\bar{t} = \frac{tc}{l}, \quad \bar{x} = \frac{x}{l}, \quad \bar{\varepsilon} = \frac{\varepsilon}{\varepsilon_s}$$

Here l is the length of rod, $c = E/\rho$ is an elastic wave velocity, ρ is a density, ε_s is a deformation, related to yield limit. Farther the dimensionless variables are used and hats above variables are droped. Initial conditions are taken as $v = \sigma = \varepsilon = 0$ at $t < 0$. Boundary conditions are taken in general forem:

$$A_1\frac{\partial v}{\partial t} + A_2 v + A_3\sigma = \phi_1(t), \quad \text{at} \quad x = 0 \tag{4}$$
$$B_1\frac{\partial v}{\partial t} + B_2 v + B_3\sigma = \phi_2(t), \quad \text{at} \quad x = 1$$

At $A_1 \neq 0$ or $B_1 \neq 0$ additional conditions are required: $t = 0$, $v|_{x=0} = v_0$, $v|_{x=1} = u_0$

§ 2. System of equations (2)—(3) has main linear part with non-differential terms? Which possibly are having jumps. This system differs from the system of equations considered in [6] only by the non-differential term with multiplier $H(\Theta)$, which is relevant to delay of plasticity. Described system of equations has following characteristic relations:

$$dx \mp dt = 0 : \; dv \mp d\sigma = \pm\delta\Phi dt,$$

$$dx = 0 : \; d\varepsilon = d\sigma + \delta\Phi dt, \tag{5}$$

where

$$\delta = \frac{e}{\tau c}, \qquad \Phi = H(\Theta)H(\kappa)\operatorname{sign}\sigma$$

Consider shocks, which may exist in medium described by Eq. (2). The system of equations (2)—(3) is devergent and it is possible to build the generalized solutions theory and to get necessary jump conditions providing uniqueness of generalized solution [7, p. 485]. Using such approach find jump conditions:

$$D[v] + [\sigma] = 0, \quad D[\sigma] + [v] = 0, \quad D[\varepsilon] + [v] = 0 \tag{6}$$

where $D = dx/dt$ is a velocity of shock propagation; $[u] = u^+ - u^-$ is a jump of u. From condition of non-trivial solution existance for system of equations (6) follows, that $D = \pm 1$, i. e. strong jump waves propagate only with elastic wave velocity. Coinsidence of shock tracks with characteristic lines allows using (5) and (6) get (same as for linear systems) differential equation for intensity of strong jump wave. But in contrast to elastic medium, where the jumps may appear only due to jumps in boundary conditions, in the case under consideration it is possible the appearance of strong shock waves even under smooth boundary conditions. This becomes possible, if for lines $dx \pm dt = 0$ the condition $\Theta = 0$ is fulfilled. For instance, when elastic wave propagates along undisturbed rod, the condition $\Theta = 0$ is fulfilled on the line $x = t - t_0$ (t_0 is the time of delay of plasticity at $x = 0$). In this case the equation for stress behind wave front $x = t - t_0$ is

$$\frac{d\sigma^+}{dt} = \delta\Phi^+ \qquad (7)$$

Because stress σ^- and deformation ε^- before wave front are constant while σ^+ is decreasing $d\sigma^+/dt < 0$, along the considered line the jump is developing: at the time instant $t = t_0$ it is equal to zero and with time its absolute value grows asimptotically tending to limiting value $|\sigma^-| - 1$. This jump propagates and reflects, collides with other jumps, arising due to boundary jumps. Created due to collisions jumps are propagating with the same velocities $D = \pm 1$ and their intensities are defined according to (6).

Remark that when the line of transfer to plastic state ($\Theta = 0$) does not coinside with lines $dx \pm dt = 0$, it is line of jump in first derivatives of solution, i. e. in this case wave of delay of plasticity will be a wave of weak jump. Lines with $\kappa = 0$ are the elastic unloading or secondary loading lines and on such lines the jump have second derivatives of solution (see. [6]). Such weak character of jumps allows to refuse from detailed consideration of multiple areas separated by elastic, plastic and plastic delay waves and to apply the through methods of calculation, accounting only one type of jumps which propagates with elastic wave velocity. Due to this the solution algorithm is simplified in comparison with algorithms based on Yu. N. Rabotnov's theory.

For numerical solution of equations (2)—(3) the following characteristic finite-difference scheme is used [6] (see Eqs. (5)):

$$(v-\sigma)^{i+1}_{j+1} - (v-\sigma)^{i}_{j+1} = \delta\Phi^{i+1/2}_{j+1} h$$

$$(v+\sigma)^{i+1}_{j+1} - (v+\sigma)^{i+1}_{j} = \delta\Phi^{i+1}_{j+1/2} h \qquad (8)$$

$$(\varepsilon-\sigma)^{i+1}_{j+1} - (\varepsilon-\sigma)^{i}_{j} = \delta\Phi^{i+1/2}_{j+1/2} h.$$

Here i and j are numbers of characteristic lines in positive and negative directions. Creating finite difference grid.

Calculations using scheme (8) are performed layer by layer along characteristic lines of positive inclination. For solving of the system of equations (8) the iterations are used on each step. As usual, it is only one iteration is required, because others practically useless. The state of material at definite grid point is checked by conditions $\Theta > 0, \Theta < 0$ и $\kappa > 0, \kappa < 0$. The scheme (8) has second order accuracy and its local stability may be proved.

So the numerical solution of problems based on proposed system of equations is appeared much better than solution of the same problems using the theory of elastoplasticity. Therefore the use of equations (2)—(3) is of sense also for materials with neglectable influence of strain rate on lower yield limit. To do this the equations for elastic plastic materials are supplied by artifitial small terms according to the law (2). Function $F(\kappa)$ and parameter $\delta = l / c\tau$ are chosen so that numerical solution tends quickly to the solution without additional terms. Asimptotic behavior of solution at $\delta >> 1$ is shown in [б] and it is pointed there that $F(\kappa) = \kappa^{1/2}$ is a good choice .

Value δ should be defined from condition of geometric similarity of solution. In dimensionless variables it means that it needs to find δ, which makes insensitive the solution to its farther growth excluding area of high solution gradients near jump lines. In practical calculations it gives the value $\delta = 50 - 100$, the change in solution is near 1 % in this case.

It should be noted that too big values of δ is not desirable because it requires to decrease the time step and hence leads to the increase of computaional work. Investigation of stability of difference scheme (8), provided for ideal plasticity stress-strain diagram indicates that the scheme is stable under condition $\delta h \le 1$. This also restricts the value δ in practical untegration.

Proposed method is used for solving problems of collided elastic plastic rods without accounting of delay of plasticity and good results are obtained [8].

§ 3. Consider some numerical results. In order to check the validness of made conclusions is considered the problem about wave propagation in rod assuming that one side of rod ($x = 1$)is fixed, while another ($x = 0$) suddenly starts to move with constant velocity $v = v_0$. Assuned that material has linear hardening:

$$f = \begin{cases} \varepsilon & \text{at} \quad \sigma \le 1 \\ k(\varepsilon - 1) & \quad\;\; \sigma > 1 \end{cases}$$

Analytical solution of this problem is given in [2].

Fig.1 shows the comparison of dependence of e in time at $x = 1$ for two solutions: solid line corresponds to numerical solution by proposed method, while the dashed line corresponds to known analytical solution [2]. Fig. 2 shows jump lines in analytical solution on the plane (x, t) for considered example. In calculations $k = 0.2$; $n = 1$; $t_1 c / e = 1$; $\delta = 64$; $v_0 = 2$; $h = 0.0025$.

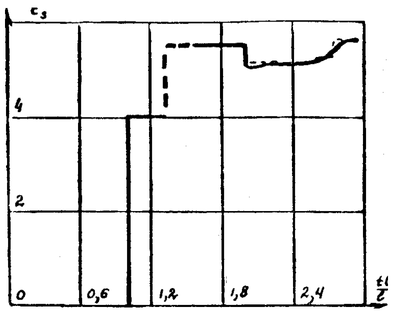

Рис. 1

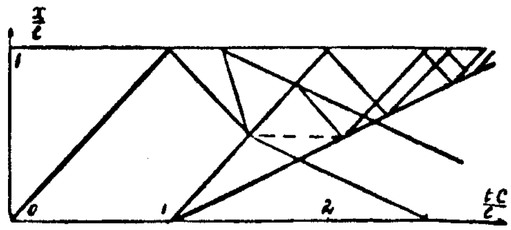

Рис. 2

It easy to see that both solutions coinside in areas with constan deformation. In stead of jumps in numerical solution we get areas of fast but smooth variation of solution. Second example is calculated for real strain-stress diagram taken for material "Сталь-3", which is approximated by piecewise-linear curve. Constants in Cotrell's delay of plasticity criterion (1) are calculated using experimental data [9] for range of delay times $10^{-1}-10^{-4}$ *sec.* It is assuned that $n=5, t_1=10^{-3.5}$ *sec.* Boundary conditions are the same as in first example, but $v_0=1.35$. Fig. 3 shows the change of deformation and stress in time at $x=0.8$. Instant jumps correspond to elastic shocks forced by delay of plasticity while the area of fast growth corresponds to plastic wave. In this example the areas of plastic deformations are appeared only after reflection of wave from fixed side of the rod in spite of the fact that the stress at another (moving) side of rod exceed the static yield limit ($|\sigma|=1.35$). The velocity of unloading wave due to delay of plasticity is almost coinsided with the velocity of elastic waves. Therefor the picture of wave motion in the plane *(x, t)* is completely different from depicted in Fig. 2.

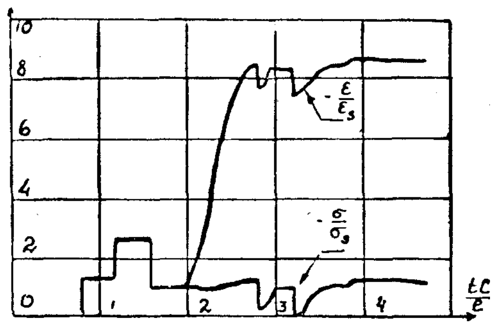

Рис. 3

References

1. Rabotnov Yu. N. Model of elastic plastic medium with delay of plasticity. «PMTF» (Journal of applied mathematics and technical physics), 1968, № 3.

2. Lomakin E. V. Propagation of elastic plastic waves in material with delay of plasticity. «Izvestia AS USSR. Solid mechanics», 1970, № 5.

3. Suvorova Yu. V. Delay od plasticity in steels (revuew of experimental studies). «PMTF» (Journal of applied mathematics and technical physics), 1968, № 3.

4. Lenskiy V. S., Fokina L. N. Propagation of one-dimensional waves in materials with delay of plasticity. «Izvestia AS USSR. Mechanics and machinary», 1959, № 3.
5. Kotlyarevskiy V. A. Elastic viscoplastic waves in material with delay of plasticity. «PMTF» (Journal of applied mathematics and technical physics), 1962, № 3.
6. Kukudzhanov V.N. Propagation of elastic plastic waves in rod with account of strain rate influence. Proc. Computer Center of AS USSR, Vol. 1. Moscow 1967.
7. Courant R. Partial difference equations. Moscow, «Mir», 1964.
8. Veklich N. A., Kukudzhanov V. N., Malyshev B. M. Duration of elastic plastic rod impact. Abstracts of IV USSR sumposium on propagation of elastic and elastic plastic waves. Kishinev, 1968.
9. Johnson G. I., Wood D. S., C1ark D. S. Delay Yielding in the Low carbon Steel under Compression Impact, Proc. Amer. Soc. for Testing Materials, 1953, vol. 53, p. 755—764.
10. Wood D. S., C1ark D. S. The Influence of Temperature upon the Time Delay for Yielding in Annealed Mild Steel. Trans. Amer. Soc. for Metals, 1952, vol. 44, p. 671—586.