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Through calculation method of fatigue damage

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Abstract. A kinetic model of damage development under cyclic loading is proposed to describe the process of fatigue failure. To determine the coefficients of the kinetic equation of damage, the well-known criterion of multiaxial fatigue failure is used. A numerical method for calculating crack-like zones up to macro fracture is proposed. The results of fatigue experiments for various geometries and asymmetry coefficients were reproduced; the operability of the model and calculation algorithm were verified.

1. Introduction

In recent decades, entire classes of criteria have been constructed that relate the number of cycles before the initiation of fatigue damage (microcracks) with the amplitudes and maximum values in the cycle (or average) that characterize the uniform stress-strain state of the working part of the specimen in a fatigue test. A large number of stress-based criteria are based on a direct generalization of the S-N Wöhler-type curves described by Basquin-type relations [1], and based upon the results of fatigue tests. Reviews on this topic are given in [2-4].

In this paper we study the processes of fatigue damage zones development using the damage theory approach dating back to [5, 6]. In the application to the cyclic loading and fatigue failure problems this approach was applied in [7, 8]. A model for the development of fatigue failure based on the evolutionary equation for the damage function is proposed. The model parameters are determined for various fatigue failure modes – low-cycle and high-cycle fatigue (LCF, HCF).

The following scheme of the amplitude fatigue curve is used. Up to value $N \sim 10^3$ the regime of repeated-static loading with amplitude slightly differing from the static tensile strength σ_B is realized. Further the fatigue curve (Wöhler curve) describes the modes of the LCF-HCF up to $N \sim 10^7$ with an asymptotic exit to the fatigue limit σ_u.

It should be noted that at present the idea of an explicit division of the classic Wöhler branch into two parts – in fact, the LCF and the HCF. The boundary of this transition region is determined not by the value of *N*, but by the value of the loading amplitude equal to the yield strength of the material σ_T [9], since this changes the physical mechanism of fatigue failure. In addition, the boundary of the repeated-static range $N \sim 10^3$ is rather arbitrary. It is also specified in [9] depending on the strength and plastic characteristics of the material. However, in this paper we will keep the suggestion of the proposed model of damage development based on the scheme described above.

In order to match the model with the well-known criteria for multiaxial fatigue failure, a stress-based criterion has been selected that describes the fatigue failure associated with the normal crack microcracks development. This is a modification of the Smith-Watson-Topper (SWT) criterion [10] described in [11], in which the amplitudes of maximum tensile stresses play a decisive role in the development of fatigue damage.

2. Kinetic equation for damage in LCF-HCF mode

The criterion of multiaxial fatigue failure in the LCF-HCF mode with the development of normal crack microcracks [11] (stress-based SWT) corresponding to the left branch of the bimodal fatigue curve (figure 1) is:

$$\sqrt{\left\langle \sigma_{1_{\max}} \right\rangle \Delta\sigma_1 / 2} = \sigma_u + \sigma_L N^{-\beta_{LH}} \tag{1}$$

where σ_1 is the largest principal stress, $\Delta\sigma_1$ is the spread of the largest principal stress per cycle, $\Delta\sigma_1 / 2$ is its amplitude. From the condition of repeated-static fracture up to values of $N \sim 10^3$ by the method [4] it is possible to obtain the value $\sigma_L = 10^{3\beta_{LH}} (\sigma_B - \sigma_u)$. According to the chosen criterion only tensile stresses lead to failure, so it includes the value $\left\langle \sigma_{1_{\max}} \right\rangle = \sigma_{1_{\max}} H(\sigma_{1_{\max}})$. In these formulas σ_B is the static tensile strength of the material, σ_u is the classic fatigue limit of the material during a reverse cycle (asymmetry coefficient of the cycle R = -1), β_{LH} is power index of the left branch of the bimodal fatigue curve.

From the fatigue fracture criterion we obtain the number of cycles before fracture at uniform stressed state:

$$N_{LH} = 10^3 \left[(\sigma_B - \sigma_u) / \left\langle \sigma_{LH} - \sigma_u \right\rangle \right]^{1/\beta_{LH}}, \quad \sigma_{LH} = \sqrt{\left\langle \sigma_{1_{\max}} \right\rangle \Delta\sigma_1 / 2} \tag{2}$$

In order to describe the process of fatigue damage development in the LCF-HCF mode, a damage function $0 \le \psi(N) \le 1$ is introduced, which describes the process of gradual cyclic material failure. When $\psi = 1$ a material particle is considered completely destroyed. Its Lame modules become equal to zero. The damage function ψ as a function on the number of loading cycles for the LCF-HCF mode is described by the kinetic equation:

$$\partial\psi / \partial N = B_{LH} \psi^{\gamma} / (1 - \psi^{\alpha})$$

where α and $0 < \gamma < 1$ are the model parameters that determine the rate of fatigue damage development. The choice of the denominator in this two-parameter equation, which sets the infinitely large growth rate of the zone of complete failure at $\psi \to 1$, is determined by the known experimental data on the kinetic growth curves of fatigue cracks, which have a vertical asymptote and reflects the fact of their explosive, uncontrolled growth at the last stage of macro fracture.

An equation for damage of a similar type was previously considered in [8], its numerous parameters and coefficients were determined indirectly from the results of uniaxial fatigue tests. In our case, the coefficient B_{LH} is determined by the procedure that is clearly associated with the selected criterion for multiaxial fatigue failure of one type or another. It has the following form.

The number of cycles to complete failure N_{LH} at $\psi = 1$ is from the equation for damage for a uniform stress state:

$$\begin{gathered} \left[\psi^{1-\gamma} / (1-\gamma) - \psi^{(1+\alpha-\gamma)} / (1+\alpha-\gamma) \right]\Big|_0^1 = B_{LH} \, N\Big|_0^{N_{LH}}, \\ N_{LH} = \alpha / (1+\alpha-\gamma) / (1-\gamma) / B_{LH} \end{gathered} \tag{3}$$

By equating the values N_{LH} from the fracture criterion (2) and from the solution of the equation for damage (3), we obtain the expression for the coefficient B_{LH} :

$$B_{LH} = 10^{-3}\left[\langle \sigma_{LH} - \sigma_u \rangle / (\sigma_B - \sigma_u)\right]^{1/\beta_{LH}} \alpha / (1+\alpha-\gamma)/(1-\gamma)$$

where the value σ_{LH} is determined by the selected mechanism of fatigue failure and the corresponding multiaxial criterion (1).

When $\sigma_{LH} \le \sigma_u$ fatigue failure does not occur, when $\sigma_{LH} \ge \sigma_B$ it occurs instantly.

3. Fatigue damage development calculation algorithm

The Ansys software was used to calculate the loading cycle of a deformable specimen, supplemented by a code to calculate the damage equation and changes of elasticity modulus.

To integrate the equation $d\psi/dN = B_{LH}\psi^{\gamma} / (1-\psi^{\alpha})$, the damage function approximation was applied at the *k*-th node of the computational grid for given discrete values ψ_k^n at moments N^n and sought ψ_k^{n+1} at moments N^{n+1}.

To calculate the damage equation, the value $\alpha = 1-\gamma$ was chosen for which by analytic integration an explicit expression for $\psi_k^{n+1}(\psi_k^n, \Delta N^n)$ can be obtained:

$$\left[\psi^{1-\gamma}/(1-\gamma) - \psi^{2(1-\gamma)}/2/(1-\gamma)\right]\Big|_{\psi_k^n}^{\psi_k^{n+1}} = B\,N\Big|_{N_n}^{N_{n+1}}$$

With the denotations $(\psi_k^{n+1})^{1-\gamma} = x$, $q = 2(1-\gamma)B\Delta N^n + (\psi_k^n)^{1-\gamma} - 2(\psi_k^n)^{2(1-\gamma)}$ and $\Delta N^n = N^{n+1} - N^n$ the equation transforms to $x^2 - 2x + q = 0$ and its valid root $x = 1-\sqrt{1-q} < 1$.The analytical expression for the increment of damage on the increment of the number of cycles ΔN^n is derived from it:

$$\psi_k^{n+1} = \left(1-\sqrt{1-\left[2(1-\gamma)B\Delta N^n + (\psi_k^n)^{1-\gamma} - 2(\psi_k^n)^{2(1-\gamma)}\right]}\right)^{1/(1-\gamma)}$$

Increment value ΔN^n defined as follows.

Based on the stress state calculation data, the coefficient B_{LH} is calculated for each node. After that, for each node, the following values are calculated

$$\Delta\tilde{N}_k^n = \left[\psi^{1-\gamma}/(1-\gamma) - \psi^{2(1-\gamma)}/2/(1-\gamma)\right]\Big|_{\psi_k^n}^{1} / B_{LH}$$

corresponding to the number of cycles at which in the node *k* from its current level of damage and equivalent stress complete destruction will be achieved (damage is equal to 1).If the damage level in the considered node is less than the threshold ψ_0 (threshold $\psi_0 = 0.95$ is selected), then the value for this node $\Delta\tilde{N}_k^n$ is multiplied by a factor of 0.5. Otherwise, it is multiplied by a factor of 1. Thus, the step of incrementing the number of cycles for a given node is $\Delta N_k^n = 0.5(1+H(\psi_k^n - 0.95))\Delta\tilde{N}_k^n$. Of all the ΔN_k^n values the smallest one is selected. The increment of the number of loading cycles for the calculation of the entire specimen is $\Delta N^n = \min\limits_k \Delta N_k^n$. For each node, based on its current level of damage and equivalent voltage, a new level of damage is found taking into account the calculated increment ΔN^n.

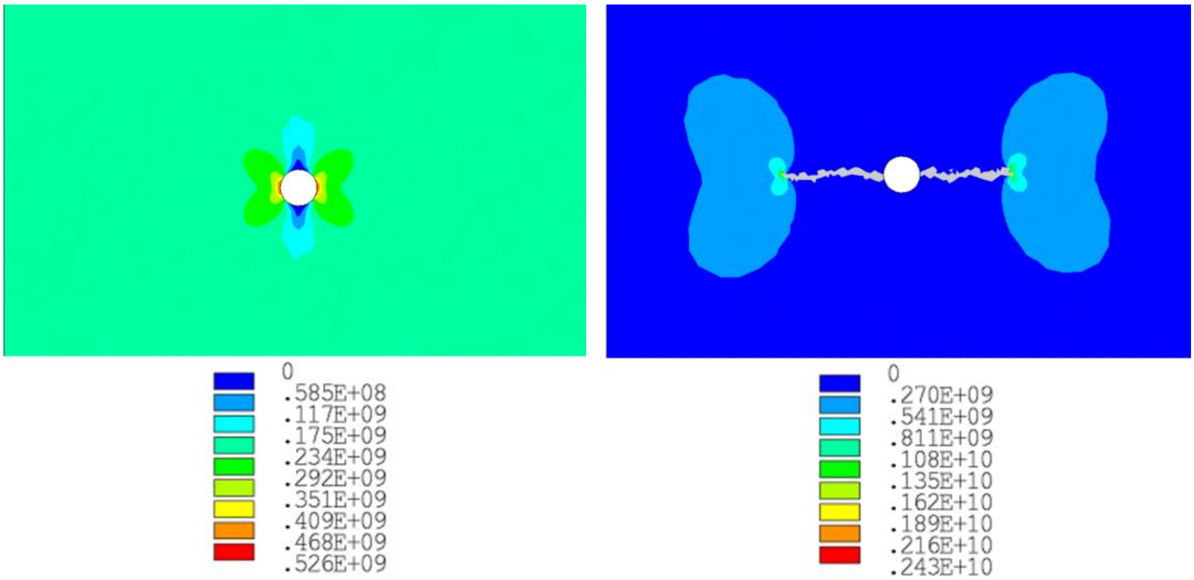
All elements are sorted out, for each of them the most damaged node is searched and according to its damage the mechanical properties of the element are adjusted:

$$\lambda(\psi_k^n) = \lambda_0(1-\kappa\psi_k^n),\ \ \mu(\psi_k^n) = \mu_0(1-\kappa\psi_k^n)$$

Those elements that belong to nodes with damage $\psi = 1$ are removed from the calculation area and form a localized zone (crack-like) of completely destroyed material. The calculation ends when the boundaries of a completely damaged region exit to the specimen surface (macro destruction) or the evolution of this region stops.

4. Calculation results

To determine parameters of the proposed model and verify its performance, one of the fatigue experiments described in [8] was performed numerically. From the condition of matching the experimental and calculated fatigue curves for a specimen of certain geometry for a given loading amplitude and cycle asymmetry the numerical coefficients were found. Using the obtained values, the experimental results on specimens of a different geometry and asymmetry coefficients were reproduced and calculation algorithm operability was confirmed.

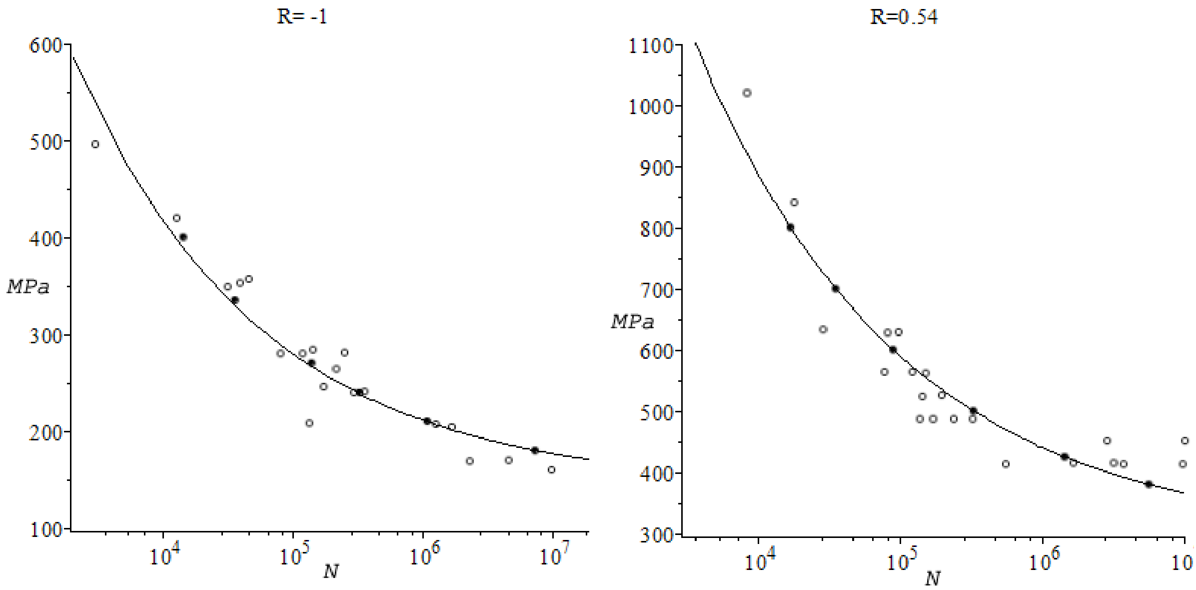


a) b)

Figure 1. Specimen with a hole at R = -1:
a) emergence of a "quasi-crack", b) growth of a "quasi-crack".

Initial tests were conducted on a plate 100x25x1.57 mm in size with 1.56 mm diameter through hole in the ones center. Ratification tests were conducted on a V-notched specimen that has 15 mm width without a notch, thickness of 1.7 mm, the notch depth of 1.32 mm, the V-notch angle of 60 degrees and the notch radius of 0.675 mm. The cyclic loading of the upper and lower boundaries of the specimen with an amplitude of 0.096 mm with the development of damage zones up to macroscopic destruction was simulated and matched with the results from [8]. In the center of the plate there is a through hole with diameter of 1.56 mm.

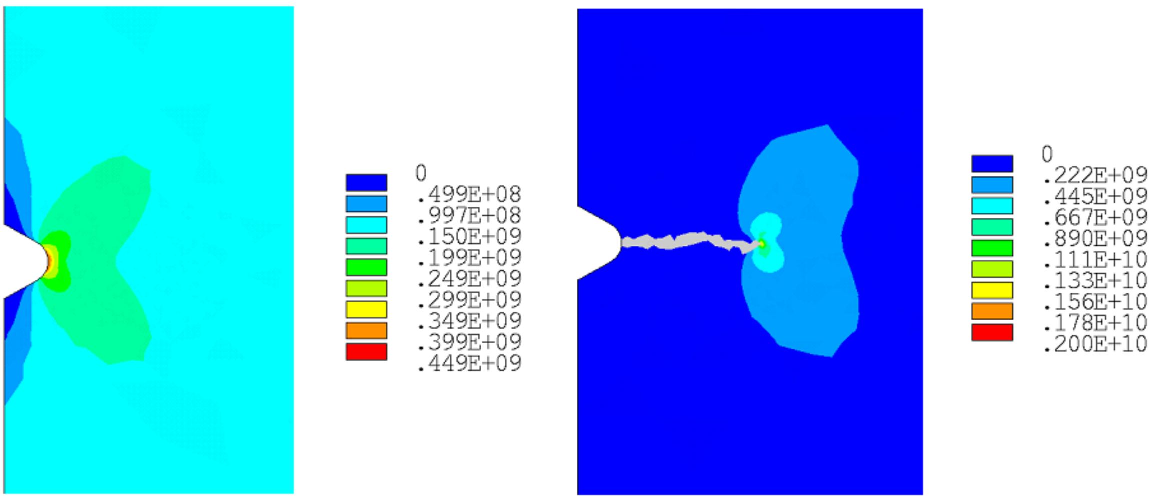
Plate material – titanium alloy with strength and fatigue parameters = 1135 MPa, = 330 MPa, = 0.31. Elasticity modulus of intact alloy are = 77 GPa, = 44 GPa. Figures 1 and 3 show the lines of the effective stress level for the specimen with a hole (figure 1) and for the specimen with a notch (figure 3) in two states – before the fatigue quasi-crack initiation and at the moment when it has passed approximately halfway to macro-destruction.



a) b)

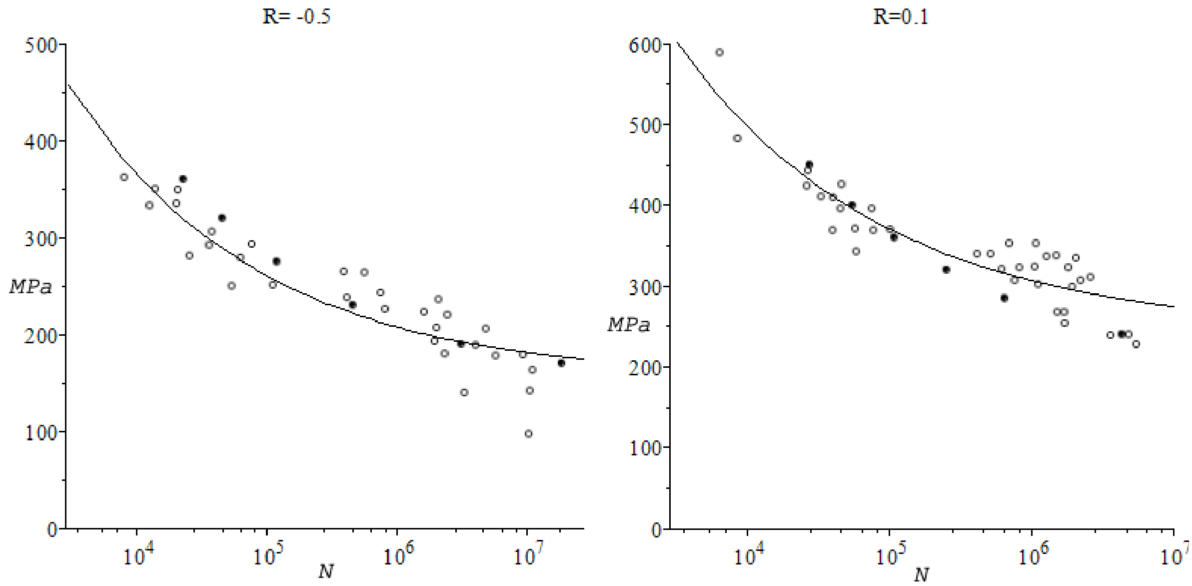
Figure 2. Fatigue curves for the specimen with a hole: ∘ - real test points, • - calculating points.

In figures 2 and 4 the results of real and computational experiments on constructing fatigue curves for specimens with a hole and a side notch are presented. Both real and calculated points represent the moment of a crack initiation. The curves in the figures approximate the experimental points. The calculations presented in figure 2-b almost exactly fit the approximation curve for the values of the model parameters. Utilizing these parameters, the fatigue curves presented in figure 2-a (specimen with a hole, R = -1) and in figure 4 (notched specimen, R = -0.5and 0.1. At figure 2-b, the calibration series, the relative error is 0. The average relative errors at figures 2-a, 4-a and 4-b are 1%, 7% and 6% respectively. The obtained satisfactory quality reproduction of real fatigue experiments indicates the efficiency and prospects of the model and calculation algorithm. The considered model represents the development of the damage model in case of cyclic loads, presented in [12] for the description of damages during dynamic loading.



a) b)

Figure 3.V-notched specimen at R = -0.5:
a) emergence of a "quasi-crack", b) growth of a "quasi-crack".



a) b)

Figure 4. Fatigue curves for V-notched specimen: ∘ - real test points, • - calculating points.

5. Conclusions

A kinetic model of cyclic loading damage development is proposed to describe the fatigue fracture process development. To determine the coefficients of the kinetic equation of damage, the known SWT criterion of multiaxial fatigue fracture was used. A numerical method for calculating crack-like zones up to macrofracture is proposed. The model parameters are determined from the condition of matching the experimental and calculated fatigue curve for a specimen of a certain geometry at a given load amplitude and cycle asymmetry coefficient. Using the obtained values, the results of experiments on specimens of a different geometry and asymmetry coefficients were reproduced and the model and calculation algorithm performance was confirmed.

Acknowledgments

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